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How the Existence of Inconsistent Solutions in Systems of Non-Homogeneous Linear Equations can be Tamed

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ABSTRACT

In this paper the issue of the long aged existence of inconsistent solutions to systems of non-homogeneous linear equations has been given another look, and some of the causes of for inconstancy in solutions were discovered. This paper has shown that inconsistent solutions are caused by an ill-posed or ill constructed equation in the system of equations, and when the ill constructed linear equation is identified and correction is effected, the system of equations will have a unique solution when the problem is solved guidelines are given on how to idetify correctly. To prove the wort of the discoveries made in this paper, example problems were taken from well known standarded text books in Linear Algebra, Tne need to tame the existence of inconsistent solutions so that students are not discouraged from having interest in mathematical problems involving non-linear systems of linear equations in future and to avoid unnecessary wast of valuable time solving problems that lead to false statements like; " 0 =5!".In a reduced echelon form when any of the equations or any of the rows have two variables that makes that particular equation to become a homogeneous system, the use of the mirror image method(MIM) makes what will have resulted to the system having infinitely many solutions to have a unique solution.

Keywords: Homogeneous , non-homogeneous systems of linear equations, inconsistent solutions, unique solutions, many solutions.

INTRODUCTION

Linear Algebra is OF great and supreme importance in sciencs,engineering fields ,management, and in real life.David Strang[1] ,gives a detailed application of linear algebra to every field of its applications.Most of the problems encountered in these areas mentioned come in forms of systems of linear equations which may be made of several variables whose values have to be determined by solving the systems of equationsin order to find their respective values.These equations may be homogeneous or non-homogeneous and at times a combination of both forms of equations.When these equations are solved ,any of these three types of solutions may arise,the solution may be:

(i)Unique,
(ii)inconsistent (no solution);
(iii) Infinitely many.

(for a homogeneous system.

Experience has shown that solutions of the form in(ii), and (iii) wll arise if any of the equations in the system is ill posed.
This seeks to prove that what causes solutions of the type of solutions in(ii),(iii),and (iv) to come up can be corrected ,so that every well posed ,or well constructed problem can have a unique solution .Most times n the

classroom ,students do ask .why do teachers wast students time solving problems yielding the type of solutions in (ii) to (iv)?.It was in answer to s

Such problems that this paper is written.

Some Definations

(a) A system of non-homogeneous linear equations Ax = b.

An example of such a system is as shown below;

$a_{11}\,x_{11} + a_{12}x_{12} +\cdots+\ a_{1n}\,x_{1n} = b_1$
$a_{21}\,x_{21} + a_{22}x_{22} +\cdots+\ a_{2n}\,x_{2n} = b_2$
……………………………….. …..
.
.
$a_{m1}\,x_{m1} + a_{m2}x_{m2} +\cdots+\ a_{mn}\,x_{mn} = b_m$

A system of homogeneous linear equations is of the form Ax=0.

Below is a system of homogeneous linear equations;

$a_{11}\,x_{11} + a_{12}x_{12} +\cdots+\ a_{1n}\,x_{1n} = 0$
$a_{21}\,x_{21} + a_{22}x_{22} +\cdots+\ a_{2n}\,x_{2n} = 0$
……………………………….. …..
.
.
$a_{m1}\,x_{m1} + a_{m2}x_{m2} +\cdots+\ a_{mn}\,x_{mn} = 0$

a_(ij^' s ,b_(i ,) are constants that are to be determined.@2 (a)Solution set (s) is a set ([s_i] of Lineaer Equations for i=1,2,…,n,which will make any linear systems of equations to be a true statement.)

Types of Solutions

(i) In a unique Solution: The system has exactly one set of values that satisfies all equations. This happens when the coefficient matrix has a non-zero determinant (for square matrices) or when there are no free variables. [1, 2]
(ii) For infinitely Many Solutions**:** The system has an endless number of solutions. This occurs when the determinant is zero and the system is consistent, leaving one or more free variables.
(iii) The case of no Solution: The system is inconsistent and has no valid answer. This happens whenon row reduction leads to a false statement like(0=5).
(iv) (iv)zero solution results in a system of linear equations when we wrongly assume that the zeros can satisfy all the equations a homogeneous $(x_{1\,,}x_n\ ) = (0, \ldots ,0\ )$

LITERATURE REVIEW

Linear Algebra has a very wide use in so many fields of endeavors because of its practical applications and the occurrence of both systems of homogeneous and non homogeneous systems of linear equations.Some of the methods used in solving systems of lniear equations include the elimination method for a two by two system of equations,the Gauss echelon method(GEM).The GEM has attracted both commendation and criticisms from both teachers and students. 2. Literature Review.

The use of the mirror-image method (MIM) was introduced by Okafor et al (2022), (2023) in balancing skeletal chemical reactions to overcome the challenges students encounter in balancing such chemical equations by the use of the Gaussian elimination method (GEM). The use of the Gaussian elimination method has attracted both admiration and criticisms from both teachers and students. Those on the side of hailing the (GEM) method include but not exclusive authors like Waed H.T (2025), when he applied the method in Fuzzy systems of Linear equations. Sefaila R.G. ((2022) gave a detailed account of the advantages and limitations of Gaussian elimination method. Sefaila and Dharmend (2025), and .Yogseeh,N.,(2016) showcased the efficiency of the application the Gaussian elimination method in the solution of complex equations of linear systems of equations.

Ichwanul et al (2024),supported the view that the GEM is eient when finding current in an electrical circuit using Krachioffs law. On the other hand, Thorne (2010),stated that the algebraic method either by the GEM ,or any method has not given relief to the less gifted in mathematics as most students find it hard solving a set of simultaneous equations., the same view held by Anton & Rorris (2014),Krisha et al(2020) explained the difficulties encountered by both students and teachers in balancing chemical equations and the use of GEM .Sanchez (2016) ,voiced it out that the use of the GEM is tedious and takes up valuable laboratory time. It was in order to circumvent the difficulties encountered in reducing a matrix to an echelon form, that Okafor et al , introduced the use the mirror-image method(MJM), and also to eliminate fractions occurring in the use of the Gaussian elimination method (GEM).

When it comes to solving systems of Homogeneous linear equations, most people have for a long time held the view that the zero-solution vector is always a solution to a homogeneous system of linear equations, Douglas (2022). If that view is upheld, then , then the only solution to the equation, $3x + 5y = 0$ will be $(x,y) = (0,0)$. However,Okafor et al (2022),and Okafor and Abraham(2026),have used the mirror-mage ir to show that the zero-solution be can eliminated completely when solving systems of homogeneouslinear equations. Many systems of linear equations have been termed to have infinitely many solutions, or no solutions, because the questions were ill conditioned or wrongly set or combined. This paper is out to lay these misleading conclusions orsolutions that lead to false statements to the dust bin by pointing out the causes of such misleading conclusions and false statements.Hardly will one find any linear algebra text book without coming to solve problems with any of thesethree options;(i) no soltion,(ii)infinitely many solutions,a (iii) zero solution..Consider book by,Seymour and Lipson (2004) [6],Davd C,Lay et al(2016),David Chemey et al (2013),[8],Gilbert Strang(2017) [13,John Hopkins book on Linear Agebra,Douglas (2020).

METHODS

Three principal methods used in this paper are:

(i) The Mirror Image Method(MIM);
(ii) The substitution method;

(the Gaussian Elimination Method (GEM).

(i) The (MIM) was introduced by Okafor et al,2022) in balancing skeletal chemical reactions, and ,in Okafor and Abraham(2026),here they used the [MM]to systems of homogeneous equations and non-homogeneous equations having one of the right han side of the equation equal to zero.
(ii) Two propositions are applicable in the MIM'

(a)Proposition 1.

If, $3x + 4y =0$.

$3x = -4y$,then. $X =4$, $y= -3$.$(x,y) = (4, -3)$.

(b) Proposition 2.

If, $3x + 4y = 0$,and $3x + 7y = 0$.

($3x = 4Y = 0$,has solution set ,$(x,y) = (4, -3)$,and $3x\ 7y = 0$,has a solution st $(x,y^{**} = (7, -3)$.To get the value of x,y that wii satisfy the two equatons ,where the value of x in the first equation is $x = 4$,and in the second equation the value of x is $x = 7$.We have the multiply the first solution of $(x,y) = (4,-3)$ by the value of x in $(x,y)^{**} = (7,-3)$,to get $7(x,y) = 7(4,-3) = (28, -21)$ .This will satisfy the two equations;

$3x + 4y = 0$,and $3x + 7y = 0$,which without the application of the MIM,should have had on lhe zero solution.

Steps on how to tame the Existence of Many Solutions,or no solution

when the solution actually exist.

When solving a problem on non -homogeneous systems of linear equations,check to see if any of the equations is

(i) A linear combination of ahy other two equations in the system
(ii) The result of subtracting one equation from the other.
(iii) Check both sides of each equation in either case to see where the mistake comes from and effect the appropriate correction.
(iv) When one of the equations in the non-homogeneous systems of linear equations is homogeneous and is of two variables that system cannot have infinitely many solutions.

EXISTENCE OF INCONSISTENT SOLUTIONS

Sheldon (2026) [12] pp has shown the condition under which EXAMPLE 1. consider this problem taken from page 21/39 Chapter 1,Systems of Linear Equations by Johns Hopkins University;
Solve a system of linear equations (Infinitelly many solutions)

$x_2 - x_3 = 0$

$x_1 - 3x_3 = -1$

$- x_2 + x_2 = 1$

Solution

$x_2 - x_3 = 0$ (1)

$x_1 - 3x_3 = -1$ (2)

$- x_1 + 3x_2 = 1$ (3)

The presence of the first equation of two variables and homogeneous in nature has shown that the solution to the system will be a unique, and not infinitely many.

Applying the MIM approach, from (1), $x_2 \quad x_3$,that is,

$(x_2\, x_3) = (1,1)$

From (2) ,

$x_1 - 3x_3 = -1$, (x_1,=2),since, $x_3 = 1$, from (1)

From (3) ,

$-x_1 + 3x_2 = 1$,

$x_1 = 2$, $x_2 = 1 = x_3$

Hence,the solution becomes $(x_1, x_{2}, x_3) = (2,1,1)$,a unique solution and not infinitely many solutions!

Example 2. Consider an example of a system of homogeneous linear equation that was thought to have infinitely many solutions but gives a unique solution when the linear combination is properly done this Example 7from the Johns Hopkins Linear algebra book (38/39).

Solve the following system using Gauss-Jordan elimination method(only one solution),

x-2y +3z =9

-x +3y =-4

2x -5y + 5z =17

Solution

Suppose that;

x-2y + 3z = 9 (1)

x+3y =-4 (2)

2x +5y +5z =1 (3)

If we change all the signs of the terms in(2) and add them to (1),we get that,

2x +5y +5z =13 (4)

Since the right hand side of (4) is less than the right hand of (2) by 4,therefore the 4 on the right hand side of (2) should be replaced by 8 to give the system of ewuations below;

x-2y + 3z = 9 (1)

x - 3y = 8 (2a

2x -5y +5z =17 (3)

Reducing to echelon form,we get

1 -2 3 9

0 -1 -5 -1

0 9 -1 -1

1 -2 3 9

0 -1 -5 -1

0 0 -46 0

The last two rows are,

-y -5z = -1

-46z =0, Y =1,Z =0

From the first equation,

$x-2y + 3z = 9$

$x-2$**=9**

$X = 11.$

Henc ,the corrected linear system of the equations has the solution (x,y,z) = (11,1,0).

The next problem from Johns Hopkins book termed to have trivial solutions is solved

$x_1 - x_2 + x_3 = 0$

$x_1 + x_2 + 3x_3 = 0$

Let the system of equations be;

$x_1 - x_2 + x_3 = 0$ (1)

$x_1 + x_2 + 3x_3 = 0$ (2)

Adding (1) ,and (2),we get;

$2x_1 + 4x_3 = 0$, therefore,

$2x_1 = -4x_3$ by the MIM,

$x_1 = 4$

$x_3 = -2$

Substituting , $x_1 = 4$, $x_3 = -2$,in (1),

$4 - x_2 - 6 = 0$

$x_2 = -2.$

Hence ,the solution ($x_1 x_2$, x_3) =(4, -2, -2),and the system has a unique solution,no free variable is needed.

We now consider problem from some of the authors mentioned in the paper.

ANALYSIS

Example 1.

Consider this problem from Lipschutz and LIPSON.PP 82-83,Example 3.12(b)

Given that,

$x_1 + x_2 - 2x_3 + 3x_4 = 4$

$2x_1 + 3x_2 + 3x_3 - x_4 = 3$

$5x_1 + 7x_2 + x_3 + x_4 = 5$

Solution

A second look at the third equation ,reveals that two times the second equation plus the first equation ,gives the left hand side of the third equation, therefore the constant number 5, on the righ hand side of the third equation as 5,should be replaced with the constant number 10 ,so the combination balances on both sides of the equation. Hence the correct system of the non-homogeneous linear equations are shown below:

$x_1 + x_2 - 2x_3 + 3x_4 = 4$ (1)

$2x_1 + 3x_2 + 3x_3 - x_4 = 3$ (2)

$5x_1 + 7x_2 + x_3 + x_4 = 10$ (3)

Line2 -2*(Line 1) , $x_2 + 7x_3 - 7x_4 = -5$ (4)

Line 3-5*line 1), $2x_2 + 11x_3 - 14x_4 = -5$ (5)

Line 5 -2*Line 4, $-3x_3 = 5$

$x_2 = \frac{20}{3}$, $x_3 = \frac{-5}{3}$, $x_4 = 0$.

Putting $x_2 = \frac{20}{3}$, $x_3 = \frac{-5}{3}$, $x_4 = 0$, ,in (1) we find that $x_1 = -6$.

The unique solution is,

$(x_1, x_2, x_3\ x_4,) = (-6, \frac{20}{3}\ , , \frac{-5}{3}\ , 0)$.

Example 2.

Find all solutions to the following system of equations by row-reducing the coefficient matrix:

$-\frac{1}{3}x_1 + 2x_2 - 6x_3 = 0$

$-\frac{1}{3}x_1 + 2x_2 - 6x_3 = 0$

$- - 4x_1 \quad -5x_3 = 0$

$3x_1 + 6x_2 - 13x_3 = 0$

$\frac{7x_1}{3} + 2x_2 - -\frac{8x_3}{3} = 0$

Solution.

This an over determined equation of four equations in three variables.

$\frac{1}{3}x_1 + 2x_2 - 6x_3 = 0$ (1)

$- - 4x_1 \quad +5x_3 = 0$ (2)

$3x_1 + 6x_2 - 13x_3 = 0$ (3)

The presence of two variables in (2) has made the reduction to echelon form unnecessary. By usung the MIM approach , we get the values of $(x_1, x_3) = (5,4)$, substituting these values in any of the three remaining equations, gives the value of $\mathbf{x_2} = \frac{2}{3}$.

Hence the complete solution of the equation becomes, $(x_1, x_2 \;\; x_3) = (5, \frac{2}{3}, 4)$.

SUMMARY

This paper has demonstrated hoe to obtain unique solutions to questions that were wrongly perceived to have either nor solution ,or infinitely many solutions when the questions were ill-condioned, or ill posed .The use of the mirror-image method was employed in solving non-homogeneous systems of linear equations to yield unique solution, once any of the equatins has a homogeneous equation.

CONCLUSION

It can be concluded that every non-homogeneous system of linear equations that has been composed will surely have a unique solution. The mirror-image method(MIM) helps to solve a singular equation of homogeneous linear equation with two variables to get a unique solution.

Conflict of Interest. The authors declare that there is no conflict of interest

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