Who Gets the AI Dividend When Housing Supply Cannot Keep Up

Kwan Hong TAN

Singapore University of Social Sciences, Singapore

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ABSTRACT

Artificial intelligence can raise productive capacity without improving the purchasing power of households that own little capital or housing. This paper develops an exchange-production model linking an AI-responsive output dividend, the income share received by renters, and housing supply. Closed-form results distinguish output growth, nominal income, housing access, and consumption-equivalent welfare. Under common Cobb-Douglas preferences, renters lose when the elasticity of their income-share decline exceeds one minus the housing expenditure share divided by one plus the housing supply elasticity. Consequently, nominal income can rise while welfare falls. An illustrative 50% output expansion raises renters' nominal income by 4.1% but reduces their welfare by 9.6% under fixed housing supply. A dividend financed by 8.9% of net housing rents restores their initial welfare in that scenario. A constant-elasticity-of-substitution extension shows how limited consumption substitution strengthens the adverse price channel. Balanced transfers leave prices unchanged under common homothetic preferences; heterogeneous housing propensities generate partial price offsets, while recipients still benefit within the model. Analytical proofs and 7,644 deterministic parameter cases verify the results. These are conditional mechanisms and numerical illustrations, not estimates of AI's observed effects. The contribution is a tractable welfare and compensation framework that connects technology distribution policy to housing capacity, with explicit fiscal limits and testable restrictions.

Keywords: artificial intelligence; housing supply; income distribution; economic rents; welfare incidence; redistribution

INTRODUCTION

An economy can become better at producing software, professional services, and other reproducible output while access to desirable housing remains scarce. For households without claims on the assets receiving the gains, the relevant question is how much of the productivity dividend survives higher living costs. A pay rise measured in money need not purchase a better consumption bundle. This paper studies that possibility without assuming unemployment, market power, or an aggregate demand collapse.

Digital technology changes several distinct economic costs, so a reduction in the cost of producing information should not be treated as an equal reduction in all consumption prices (Goldfarb & Tucker, 2019). AI also changes who receives income. Acemoglu and Restrepo (2019) distinguish task displacement from the creation of new tasks, while Acemoglu (2025) explains why economy-wide gains depend on the tasks affected and the savings actually achieved. These mechanisms motivate an output dividend and an income-share response, but do not establish their magnitudes for the present model.

Housing provides a concrete setting in which productive gains encounter a separate supply constraint. Saiz (2010) links housing supply elasticity to geographical and regulatory constraints. Moretti (2013) shows that nominal wage comparisons can differ materially from comparisons adjusted for local living costs. The economic issue is therefore more specific than whether AI increases inequality: under what conditions can a household receive a larger nominal income and nevertheless lose from a productivity expansion, and what transfer would prevent that loss?

The analysis represents renters and asset owners as two fixed household groups. The economy receives an exogenous increase in nonhousing output, while the share accruing to renters may decline. A housing sector converts some of that output into housing services and earns net rents on scarce capacity. All housing rents accrue initially to the owner group. Goods markets and household budgets clear jointly, including the feedback from housing rents to owners' expenditure. This closure prevents the common accounting error of treating rent payments as if they disappeared from the economy.

Three linked results organize the paper. First, a welfare threshold combines income-share erosion with housing supply elasticity. Second, a compensation formula states the fraction of net housing rents needed to preserve renters' baseline welfare and identifies cases where that fiscal base is insufficient. Third, a transfer-incidence result separates the common-preference case, in which balanced redistribution does not move the housing price, from heterogeneous spending propensities, in which redistribution can increase it. An extension replaces fixed expenditure shares with constant-elasticity-of-substitution preferences and derives an exact local welfare criterion.

The contribution is an integration and extension of established mechanisms. It does not claim that the capitalization of productivity into scarce assets, or the difference between nominal and real income, is newly discovered. A closely related preprint by Song (2026) studies AI automation, expenditure reallocation, and fixed-asset ownership. The present model instead makes housing supply elastic, specifies the path of renters' income shares, and derives finite-change compensation requirements together with a supply-and-substitution welfare threshold. These differences define the paper's narrower novelty claim.

RELATED LITERATURE AND RESEARCH POSITION

Productivity and the distribution of income

The task approach to automation explains why higher productivity and a lower labour share can coexist (Acemoglu & Restrepo, 2019). Acemoglu (2025) further cautions against translating task-level capabilities mechanically into large aggregate gains. Here, the output multiplier is a scenario variable, and the income-share response is a reduced-form parameter. Neither is estimated from current AI adoption. The framework can therefore compare labour-complementing and labour-displacing outcomes without embedding a forecast about which will prevail.

The income share in this paper is broader than the national labour share. It is the fraction of nonhousing output accruing to the particular renter group under examination. Wages, business income, and dispersed ownership of AI capital can all enter that share. This distinction matters because many workers own assets, and many renters receive income beyond wages. Empirical implementation must measure the group's complete income exposure instead of equating tenure with a single factor of production.

Housing and the conversion of income into welfare

Housing affects both expenditures and asset income. Rognlie (2015) emphasizes housing's role in the evolution of the net capital share. Moretti (2013) distinguishes nominal wage inequality from inequality adjusted for living costs, while Diamond (2016) studies the welfare implications of workers' differing location choices. These contributions motivate a welfare measure that includes housing prices and a clear separation between renters and recipients of housing rents.

Hsieh and Moretti (2019) examine how housing constraints can restrict access to productive locations. Their quantitative conclusions have been challenged by Greaney (2026), who identifies coding and scale-dependence problems and proposes a modified model. The present paper uses this debate to motivate transparent normalization and reproducibility, and imports none of those disputed numerical estimates. Its closed economy has no migration or location choice, so it cannot quantify spatial misallocation.

AI scarcity and the boundary of the contribution

Song (2026) directly connects cheaper automated production to scarcity rents and access, with an expenditure condition governing whether rents rise. That work also shows why the distribution of asset ownership matters. The present paper takes a different primitive: a realized increase in available nonhousing output, rather than an automated-good price reduction whose expenditure response must be solved separately. The distinction is substantive. A fall in an AI service price does not by itself identify the output multiplier used here.

Related work by Tan examines how fixed assurance costs affect AI adoption and market concentration (Tan, 2026a), and how correlated agent failures can create external costs (Tan, 2026b). The current paper concerns household incidence after an output dividend is realized. The housing price channel is a pecuniary redistribution within the model; it is not the technological externality studied in the latter paper. Consequently, the proposed transfer implements a specified distributional objective and is not presented as a Pigouvian correction.

The analytical question left for this paper is the joint role of housing supply, consumption substitution, and recipients' income exposure in determining the size and fiscal feasibility of compensation. Its empirical contribution is a set of restrictions suitable for future testing. It provides no new observational estimate of AI, wages, rents, or household welfare.

METHODOLOGY AND ECONOMIC ENVIRONMENT

Research design and scope

The study combines analytical modelling with deterministic numerical evaluation. Propositions are derived from household optimization, housing production, and market clearing. Numerical exercises verify the identities, illustrate economically distinct cases, and examine alternative preferences. There are no sampled households, synthetic microdata presented as observations, fitted coefficients, or significance tests. The parameter grids are designed scenarios; their frequencies have no population interpretation.

The economy contains one nonhousing consumption composite, whose price is normalized to one, and one housing service. Two internally homogeneous groups have fixed populations: renters, indexed by r, and owners, indexed by o. Quantities and incomes below are group aggregates. With fixed group sizes and identical preferences within each group, proportional changes in aggregate indirect utility equal proportional changes for a representative member. Housing quality and the boundaries of the relevant market are held constant.

Let $Y_0 > 0$ be baseline available output and $g \geq 1$ its multiplier. A technological improvement, including AI adoption, can generate this dividend, but the model does not specify its production function. Output is measured after the resource requirements of producing the nonhousing composite, including AI operating inputs, but before the housing sector uses that composite. Renters receive the share $s(g)$ of output before housing rents and redistribution:

$$Y = gY_0, \qquad s(g) = s_0 g^{-\gamma}, \qquad 0 < s_0 < 1, \quad \gamma \geq 0. \tag{1}$$

The share-erosion elasticity γ summarizes how the dividend is distributed. Nominal pretransfer renter income rises if $\gamma < 1$, is constant if $\gamma = 1$, and falls if $\gamma > 1$. Owners receive the remaining output. The domain keeps shares feasible for all $g \geq 1$. A rising renter share can be studied locally, but is not needed for the loss conditions.

Housing technology and resource accounting

Let $p > 0$ be the price of housing services, H their supply, $\kappa > 0$ a capacity parameter, and $\eta \geq 0$ the supply elasticity. For positive elasticity, aggregate variable resource cost is the integral of inverse supply:

$$H = \kappa p^{\eta}, \qquad C(H) = \frac{\eta}{1+\eta} \kappa^{-\frac{1}{\eta}} H^{\frac{1+\eta}{\eta}} \quad (\eta > 0). \tag{2}$$

For $\eta = 0$, the economy instead has a fixed endowment $H = \kappa$ with zero variable cost. This is a separate limiting specification, so the cost formula containing division by η is never evaluated at zero. Costs represent nonhousing resources used to supply housing services over the model's horizon. They are not purchase prices of existing houses.

Write gross housing expenditure as $R = pH$. Net housing rents, denoted Π, are revenues after variable resource costs:

$$R = pH, \qquad C = \frac{\eta}{1+\eta} R, \qquad \Pi = R - C = \frac{R}{1+\eta}. \tag{3}$$

These rents can be interpreted as returns to scarce sites or other quasi-fixed housing capacity under competitive supply. The model excludes new fixed investment costs; in an application, required replacement and financing costs must be distinguished from genuinely taxable economic rents. They do not require landlord monopoly power. All such capacity belongs initially to owners. A policy pays renters the fraction τ of net rents, financed by an equal deduction from owners, where $0 \leq \tau < 1$.

The resulting group incomes are

$$I_r = sY + \tau\Pi, \qquad I_o = (1-s)Y + (1-\tau)\Pi. \tag{4}$$

The levy is on net rents after full variable-cost deduction. At a fixed tax rate below one, multiplying housing profit by $1 - \tau$ leaves the output choice unchanged in this static model. The result does not apply automatically to a tax on gross rental revenue, structures, or capital investment. A 100% rate is an accounting bound; at that endpoint the profit-maximizing supply incentive becomes indeterminate.

Household demand and welfare

The benchmark uses the same normalized Cobb-Douglas utility for each group, with housing expenditure share $0 < \theta < 1$:

$$U_i = \left(\frac{c_i}{1-\theta}\right)^{1-\theta} \left(\frac{h_i}{\theta}\right)^{\theta}, \qquad c_i + ph_i \leq I_i. \tag{5}$$

Here c_i and h_i denote nonhousing and housing consumption. Optimization gives

$$c_i = (1-\theta)I_i, \qquad h_i = \frac{\theta I_i}{p}, \qquad V_i = \frac{I_i}{p^\theta}. \tag{6}$$

The normalization makes p^θ the exact expenditure index. Ratios of V_i across equilibria have a consumption-equivalent interpretation: at baseline prices they equal ratios of the expenditure required to attain the two utility levels. They are not comparisons of cardinal utility across different people.

Market clearing requires total housing demand to equal H and nonhousing consumption plus housing resource costs to equal Y:

$$h_r + h_o = H, \qquad c_r + c_o + C = Y. \tag{7}$$

Household income sums to $Y + \Pi$, not Y, because housing capacity earns income. Final nonhousing consumption is $Y - C$. Adding housing expenditure gives the accounting identity $Y - C + R = Y + \Pi$. Thus Y is available nonhousing output, not a measure of total real GDP. Endogenous housing prices make nominal income aggregates especially unsuitable as welfare proxies.

Table 1 summarizes the principal quantities.

Table 1. Principal model quantities

Symbol	Economic meaning	Domain or unit
g	Multiplier of available nonhousing output	At least 1
$s_0,\ s(g)$	Renters' initial and current shares of that output	Between 0 and 1
γ	Elasticity of the decline in renters' output share	Nonnegative
θ	Common housing budget share in the benchmark	Between 0 and 1
$\eta,\ \kappa$	Housing supply elasticity and capacity	Nonnegative; positive
$p,\ H$	Housing service price and quantity	Numeraire per service; services
$R,\ C,\ \Pi$	Gross rent, resource cost, and net housing rent	Numeraire
$\tau,\ T$	Redistributed share of net rents and transfer amount	Rate; numeraire
σ	Consumption substitution elasticity in the extension	Positive

ANALYTICAL RESULTS

Equilibrium and the renter welfare threshold

Proposition 1. Under common Cobb-Douglas preferences, the positive goods-market equilibrium is unique. Housing expenditure, price, and net rents satisfy

$$R = \frac{\theta(1+\eta)Y}{1+\eta-\theta}, \qquad p = \left(\frac{\theta(1+\eta)Y}{\kappa(1+\eta-\theta)}\right)^{\frac{1}{1+\eta}}, \qquad \Pi = \frac{\theta Y}{1+\eta-\theta}. \tag{8}$$

Aggregate demand implies $R = \theta(Y + \Pi)$. Substitution from Equation (3) gives the expenditure expression; the supply equation then determines a unique positive price. These prices are independent of s and τ because redistributing a given aggregate income does not change aggregate demand under identical homothetic preferences. Appendix A verifies resource clearing.

For comparisons with the untaxed baseline, define $q_r = \frac{V_r(g,0)}{V_r(1,0)}$ and let ε denote the welfare elasticity when the renter share is constant. Equation (8) implies

$$\varepsilon = 1 - \frac{\theta}{1+\eta}, \qquad q_r = g^{\varepsilon-\gamma}. \tag{9}$$

Proposition 2. Renters lose from any expansion $g > 1$ if and only if $\gamma > \varepsilon$. A nominal income increase and a welfare decline coexist precisely when

$$1 - \frac{\theta}{1+\eta} < \gamma < 1. \tag{10}$$

Housing access gives a different comparison. With no transfer, the ratio of renters' housing consumption to baseline is $g^{\frac{\eta}{1+\eta}-\gamma}$. It is therefore possible to lose housing access while gaining overall consumption-equivalent welfare. The results should not be interpreted as making those outcomes interchangeable.

The benchmark also provides an explicit supply requirement. For $0 \le \gamma < 1$, renter welfare is nondecreasing in the productivity comparison whenever

$$\eta \ge \max\left(0, \frac{\theta}{1-\gamma} - 1\right). \tag{11}$$

No finite supply elasticity guarantees a gain when $\gamma \geq 1$: under those conditions renter nominal income does not grow. Under constant income shares, by contrast, benchmark welfare always rises because $\varepsilon > 0$. Scarce housing alone is insufficient for welfare losses in this specification. Figure 1 plots the threshold rather than presenting losses as an inevitable consequence of AI.

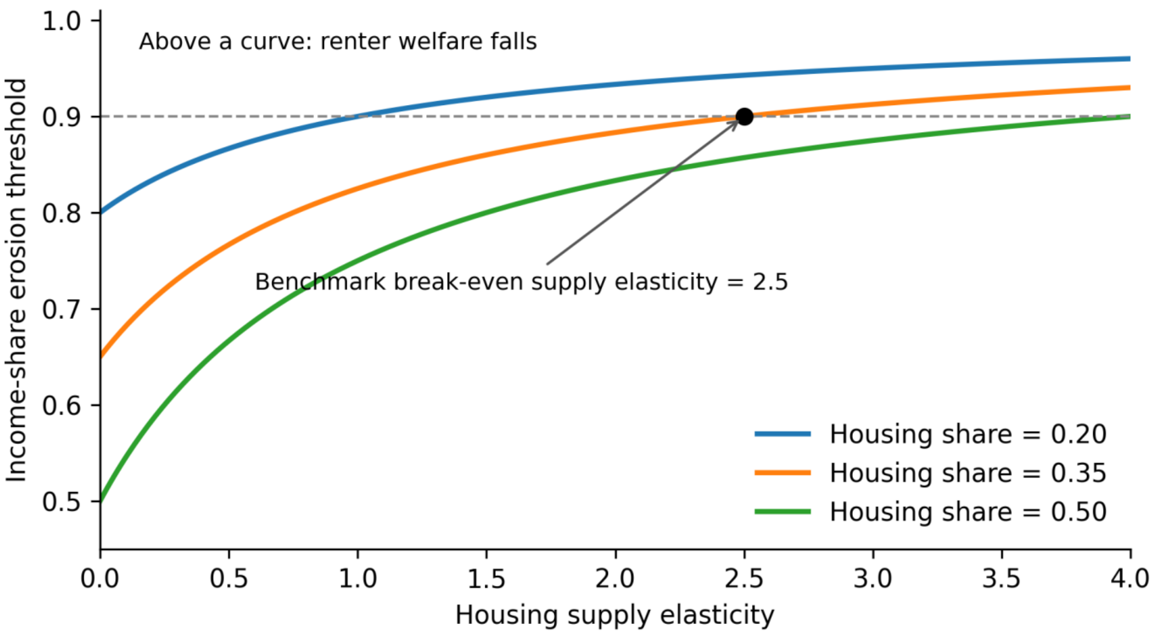

Figure 1. The income-share erosion threshold depends on housing supply. Curves show Equation (9) for three housing shares. Points above a curve imply falling renter welfare. The horizontal dashed line marks the illustrative erosion elasticity of 0.90. These are analytical boundaries, not estimated relationships.

Compensation and its fiscal limit

Proposition 3. Under common Cobb-Douglas preferences, the minimum rate on net housing rents that restores baseline renter welfare has the accounting value

$$\tau_{\min} = \frac{1+\eta-\theta}{\theta} \max\left(0,\ s_0 g^{-\varepsilon} - s(g)\right). \quad (12)$$

A value below one is implementable within the stated tax technology. A value above one means housing rents alone cannot finance compensation. Equality to one reaches the limiting fiscal bound and requires an implementation that preserves supply incentives. The proof follows by imposing $\frac{I_r}{p^\theta} \geq V_r(1,0)$ and using Equation (8). No increase in measured output is mistaken for additional government revenue.

The corresponding minimum transfer relative to output is especially simple:

$$\frac{T_{\min}}{Y} = \max\left(0,\ s_0 g^{-\varepsilon} - s(g)\right). \quad (13)$$

Holding the initial price normalization and other parameters fixed, a larger η lowers the required transfer amount because it moderates the housing price increase. The required tax rate need not decline monotonically over every parameter region: more elastic supply also reduces the net-rent base. Fiscal policy should therefore report both the transfer amount and the fraction of rents needed to finance it.

Compensation is a distributional target, not a characterization of the welfare-maximizing tax. The model has no social welfare weights, administrative costs, or dynamic distortion estimates. It also does not count a transfer twice as both a benefit to renters and a new resource for the economy. Owners finance the payment; their welfare must be evaluated separately.

NUMERICAL RESULTS AND VERIFICATION

An illustrative output dividend

Normalize $Y_0 = 1$ and the baseline housing price to one. For each supply elasticity, set $\kappa = \frac{\theta(1+\eta)}{1+\eta-\theta}$. This aligns initial prices across scenarios while allowing baseline housing quantities to differ. Cross-scenario results compare proportional changes within each economy; they do not measure the causal effect of changing a city's supply elasticity while holding its entire initial allocation fixed.

The illustration uses $s_0 = 0.65$, $\theta = 0.35$, $\gamma = 0.90$, and $g = 1.50$. These values expose a mechanism rather than fit a country. In particular, a 50% expansion is a stress scenario with no assigned date or probability. It is not a projection of the effects of currently available AI. A share-erosion elasticity of 0.90 reduces the renter output share from 0.6500 to 0.4513, although the group's nominal income rises by 4.14%.

Table 2. Housing supply and the incidence of the same output dividend

Supply elasticity	Housing price change %	Renter welfare change %	Minimum net-rent rate %	Transfer as % of output
0.00	50.00	-9.64	8.94	4.81
0.50	31.04	-5.26	8.24	2.51
1.00	22.47	-3.00	6.57	1.39
2.00	14.47	-0.67	2.32	0.31
2.50	12.28	0.00	0.00	0.00
4.00	8.45	1.22	0.00	0.00

Note. In all rows, available output rises by 50% and pretransfer renter income by 4.14%. Welfare is relative to each economy's own untaxed baseline. Tax rates apply to net housing rents, not gross rental expenditure or output. Entries are rounded from the replication output.

Table 2 makes the distinction between productive capacity and household purchasing power concrete. With fixed housing, the cost-of-living index rises by 15.25%, more than the 4.14% income gain, producing a 9.64% welfare loss. A payment equal to 4.81% of nonhousing output restores baseline renter welfare. It requires 8.94% of net housing rents because that rent base is relatively large under fixed supply. Owners' consumption-equivalent welfare remains 52.22% above their own baseline after that payment. This establishes a feasible improvement for both groups in this particular example, not for every parameter combination.

At a supply elasticity of 2.50, the welfare effect is exactly zero. Above that boundary, the price response is sufficiently muted for the small nominal income gain to improve welfare. The housing quantity received by renters can still decline: the benchmark prioritizes overall expenditure-equivalent welfare rather than a guarantee of unchanged housing consumption. Figure 2 shows how the loss and compensation rate vary with the size of the output dividend.

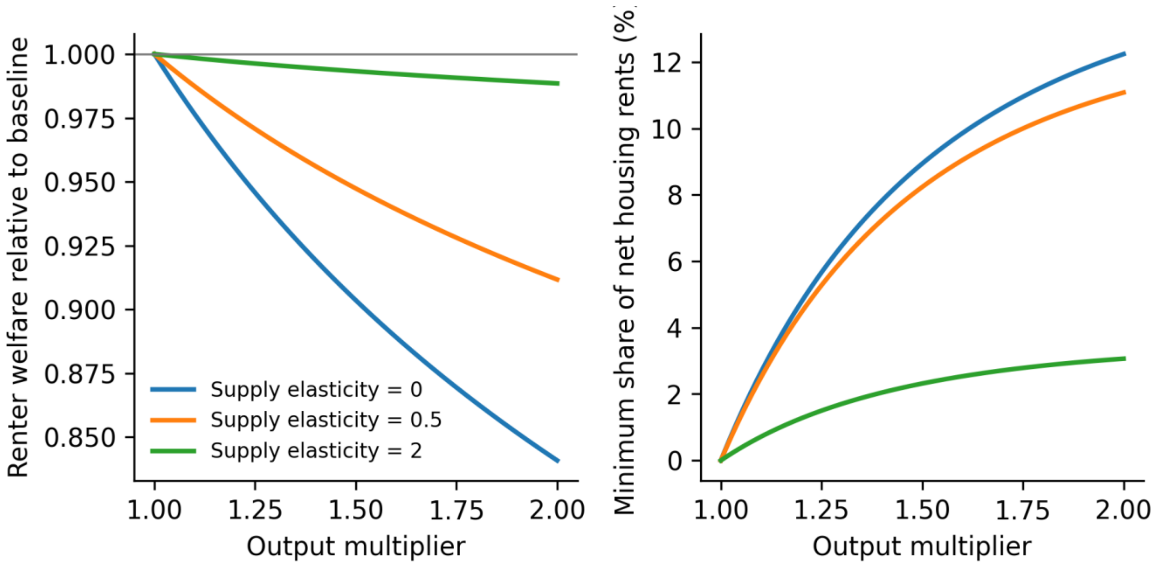

Figure 2. Welfare losses and compensation along an output expansion. Both panels use $s_0 = 0.65$, $\theta = 0.35$, and $\gamma = 0.90$. The left panel reports untaxed welfare relative to baseline; the right reports the minimum fraction of net housing rents needed for compensation. Line colours identify the same supply elasticities in both panels.

Designed parameter checks

The main grid evaluates every combination of five housing shares, six supply elasticities, seven share-erosion elasticities, seven output multipliers, and four initial renter income shares, giving 5,880 cases. Of these designed cases, 2,100 exhibit falling renter welfare and 420 combine that loss with rising nominal income. Seventeen require more than the entire net-rent base for compensation. These infeasible cases are retained in the outputs rather than clipped to a tax rate of one. These counts describe the chosen grid; they do not estimate how commonly any outcome occurs in real economies.

An additional 1,440 cases examine the substitution extension in Section 7, and 324 examine the heterogeneous-preference transfer mechanism in Section 6. Every case satisfies the relevant housing and nonhousing resource identities within a relative tolerance of 10^{-10}. The largest observed relative clearing residual is below 5×10^{-14}. The local welfare derivative in the substitution extension agrees with centred numerical differences to within 3×10^{-10}. Appendix B specifies every grid, and the accompanying code regenerates the figures, tables, and numerical audit.

REDISTRIBUTION WHEN HOUSING PREFERENCES DIFFER

Endogenous prices under a balanced transfer

The price invariance in Proposition 1 relies on common preferences. To relax it transparently, allow different Cobb-Douglas housing propensities θ_r and θ_o, with $0 < \theta_o \leq \theta_r < 1$. Define $\Delta = \theta_r - \theta_o$ and and $\omega = \theta_o + \Delta s$. For a balanced transfer T from owners to renters, housing expenditure is

$$R = \frac{(1+\eta)(\omega Y + \Delta T)}{1+\eta-\theta_o}. \qquad (14)$$

The price is $\left(\frac{R}{\kappa}\right)^{\frac{1}{1+\eta}}$. . With $T = \tau\Pi$,, the feedback between redistribution and the tax base closes exactly:

$$\Pi = \frac{\omega Y}{1+\eta-\theta_o-\Delta\tau}, \qquad T = \tau\Pi. \qquad (15)$$

The denominator is strictly positive for admissible parameters, including the limiting accounting case $\tau = 1$, because it is then $1 + \eta - \theta_r > 0$.. Redistributing toward a group with a higher housing propensity raises housing demand and its price. For identical propensities, the price effect is zero. If the recipients have a lower propensity, the sign reverses. Income rank by itself does not identify the sign.

6.2 A bound on the price offset

Proposition 4. For $\theta_r \geq \theta_o$ and a feasible balanced transfer, recipients gain in consumption-equivalent terms. The marginal purchasing-power retention from one additional unit of transfer, accounting for the equilibrium price response, is

$$\mathscr{M} \equiv p^{\theta_r} \frac{dV_r}{dT} = 1 - \frac{\theta_r}{1+\eta} \frac{\Delta(sY+T)}{\omega Y + \Delta T}. \quad (16)$$

Since $\omega Y + \Delta T - \Delta(sY + T) = \theta_o Y > 0$, it follows that

$$1 - \frac{\theta_r}{1+\eta} < M \leq 1. \quad (17)$$

The price offset is $1 - \mathscr{M}$. It is the reduction in the recipient's marginal money-metric benefit, evaluated at the prevailing price. It is not the fraction of each transfer mechanically received by landlords, nor a measure of deadweight loss. Equation (17) rules out complete erosion of the recipient benefit within this model, even under fixed housing supply. Along the net-rent policy, Trises monotonically with τ, so renter welfare also rises with the tax rate.

Table 3. Redistribution with different housing propensities

Supply elasticity	Net-rent rate %	Housing price change %	Renter welfare change %	Marginal price offset %
0	10	2.74	9.02	12.79
0	25	7.14	23.1	14.12
0	50	15.38	48.26	16.32
0.5	10	1.08	5.63	8.29
0.5	25	2.76	14.33	8.82
2	10	0.24	2.66	4.05
2	25	0.61	6.7	4.17

Note. $Y = 1.5$, $s = 0.45$, $\theta_r = 0.45$, $\theta_o = 0.25$, and $\kappa = 1$. CChanges are relative to the no-transfer equilibrium at the same output and supply parameters, not relative to the pre-expansion economy. The final column is $100(1 - \mathscr{M})$, evaluated at the indicated transfer.

Table 3 illustrates why policy evaluation must distinguish the housing price from recipient welfare. A 25% rate on net rents raises the price by 7.14% with fixed supply, yet renter welfare rises by 23.10%. Recipients face a higher price but have sufficient additional income to gain. Increasing supply elasticity reduces the price offset; at a fixed tax rate it also shrinks the rent base, so the transfer and the renter gain can be smaller. This comparison does not show that constrained supply is desirable. The heterogeneous-preference scenario uses $s = 0.45$ independently of the income-share path in Table 2 and tests transfer incidence at a fixed output level.

CONSUMPTION SUBSTITUTION AND THE LIMITS OF THE BENCHMARK

A general homothetic extension

Cobb-Douglas preferences impose unit substitution elasticity and constant expenditure shares. Replace them with a common constant-elasticity-of-substitution expenditure index, retaining the same income distribution and housing technology. For $\sigma > 0$ and $\sigma \neq 1$, define

$$P(p) = \left((1-\theta) + \theta p^{1-\sigma}\right)^{\frac{1}{1-\sigma}}, \qquad b(p) = \frac{\theta p^{1-\sigma}}{(1-\theta)+\theta p^{1-\sigma}}. \tag{18}$$

Here $b(p)$ is the housing expenditure share and $V_i = \frac{I_i}{P(p)}$. The corresponding CES utility has weights $(1-\theta)^{\frac{1}{\sigma}}$ and $\theta^{\frac{1}{\sigma}}$ on the two quantity terms. At $\sigma = 1$, the continuous limit is $P(p) = p^{\theta}$ and $b(p) = \theta$, recovering the benchmark. The preference parameter θ equals the housing share at $p = 1$, but not generally elsewhere.

Proposition 5. With common CES preferences, the positive goods-market equilibrium exists and is unique. Its housing price solves

$$Y = \kappa\left(\frac{1-\theta}{\theta} p^{\eta+\sigma} + \frac{\eta}{1+\eta} p^{1+\eta}\right). \tag{19}$$

The right-hand side is strictly increasing from zero to infinity. This proves existence and uniqueness directly, including fixed supply. Common homothetic preferences also preserve the price invariance of balanced transfers: aggregate demand depends on aggregate income rather than its distribution.

Implicit differentiation gives the housing-price elasticity $\varepsilon_p$and the renter welfare derivative at zero transfer:

$$\varepsilon_p \equiv \frac{d\ln p}{d\ln Y} = \frac{1+\eta-b}{(1+\eta)\left(\eta+\sigma(1-b)\right)}, \qquad \frac{d\ln V_r}{d\ln Y} = 1-\gamma-b\varepsilon_p. \tag{20}$$

Thus the local loss condition is $\gamma > 1 - b\varepsilon_p$. Under Cobb-Douglas preferences this reduces to Proposition 2. Under fixed housing supply, $\varepsilon_p = \frac{1}{\sigma}$. Limited substitution can make the price burden substantially stronger than in the benchmark; sufficiently high housing shares can even produce a local loss with constant renter income shares. This is a property of the extension, not a claim about empirically measured elasticities. If $\eta = 0$ and $\gamma = 0$, the exact local loss condition is $b > \sigma$. Thus constant income shares rule out losses in the Cobb-Douglas benchmark but not across the broader CES family. The condition is impossible when $\sigma \geq 1$, since $b < 1$.

7.2 Finite changes and compensation

The CES threshold is local because $b(p)$ changes along an expansion. Finite changes should be computed from the exact price index. With the normalization $p_0 = 1$, the corresponding welfare ratio and compensation requirement are

$$q_r^{\text{CES}} = \frac{g^{1-\gamma}}{P(p_g)}, \qquad \tau_{\min}^{\text{CES}} = \max\left(0, \frac{s_0 Y_0 P(p_g) - s(g)Y}{\Pi_g}\right). \tag{21}$$

As before, a required rate above one exceeds the available housing-rent base. Equation (21) makes clear why a local elasticity should not be extrapolated mechanically over a large productivity change.

Table 4. Substitution and supply jointly determine the welfare effect

Supply elasticity	Substitution elasticity	Housing price change %	Renter welfare change %	Minimum net-rent rate %
0	0.5	125	-24.57	18.2
0	1	50	-9.64	8.94
0	2	22.47	-2.55	2.69
0.5	0.5	45.44	-9.4	13.15

0.5	1	31.04	-5.26	8.24
0.5	2	18.76	-1.62	2.83
2	0.5	16.63	-1.45	4.77
2	1	14.47	-0.67	2.32
2	2	11.43	0.4	0

Note. $s_0 = 0.65$, $\theta = 0.35$, $\gamma = 0.90$, and $g = 1.50$. Baseline prices equal one. The unit-substitution rows reproduce Table 2. Every row describes an illustrative economy rather than an estimated parameterization.

The stronger losses under limited substitution in Table 4 identify a central empirical requirement: a credible application must estimate how households substitute when housing becomes more expensive. The sign is not robust to every preference and supply combination. With supply elasticity 2 and substitution elasticity 2, renters gain slightly in the benchmark expansion. Figure 3 displays the fixed-supply comparison over the full plotted expansion.

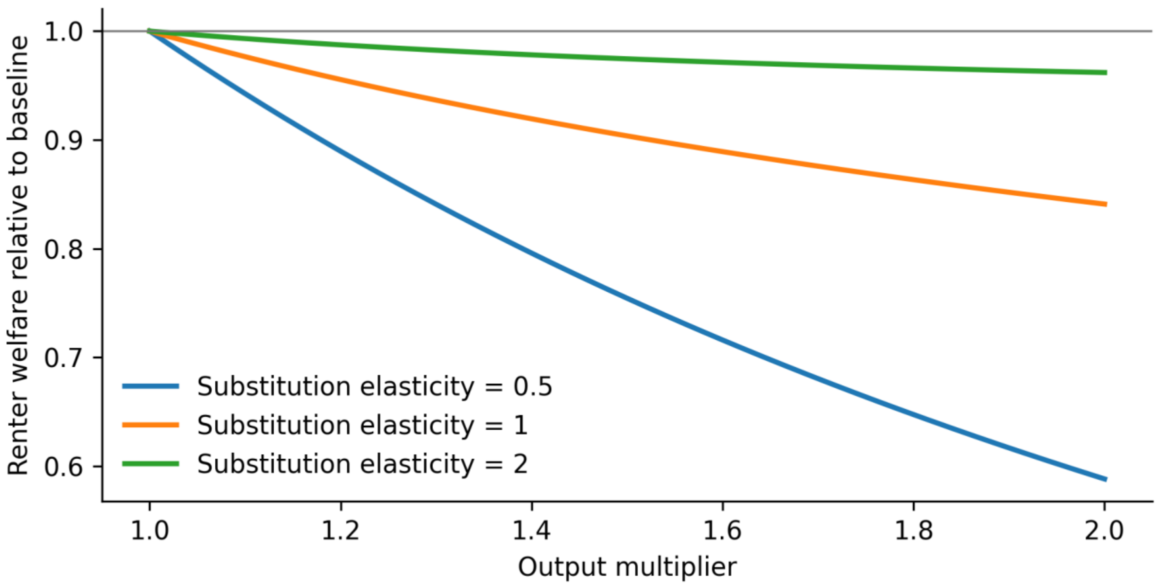

Figure 3. Limited substitution strengthens the housing price channel. Fixed housing supply and the same income-share path are combined with three substitution elasticities. Welfare is calculated from the exact CES expenditure index. The unit-elastic case reproduces the fixed-supply line in Figure 2.

DISCUSSION AND AN EMPIRICAL RESEARCH AGENDA

Evaluating the AI dividend

The model separates four outcomes that are often discussed as if they moved together: available output, nominal household income, access to housing services, and consumption-equivalent welfare. In the main illustration, the first two rise and the latter two fall for renters. This does not imply that society has fewer resources. It means that ownership and relative prices determine how a larger resource envelope reaches a particular group.

The practical measurement target is the joint distribution of technology-related income and housing exposure. Renters with diversified financial claims may receive a larger share of the dividend than the model's most exposed group. Owner-occupiers receive housing services as well as bearing housing costs; pension funds and public housing providers further distribute property income. The two-group model should therefore be used to identify incidence channels, not to classify every renter as disadvantaged or every owner as a net beneficiary.

The housing supply parameter also has a horizon. A short-run stock can be nearly fixed even where longer-run construction is responsive. Policy may therefore confront an immediate income problem and a slower capacity response. The static framework compares horizons through alternative elasticities but does not solve the transition between them. It cannot determine the optimal timing of transfers or development expenditure.

What the policy results support

Under the stated net-rent tax technology, redistribution can preserve a vulnerable group's welfare without directly taxing the AI output increase. The compensation formula provides a transparent fiscal target. It also identifies when the selected revenue base is inadequate, in which case other income sources, broader capital ownership, or additional housing capacity would be required to meet that target. No result establishes a universal basic income level, an optimal land tax, or an actual country's tax capacity.

Supply expansion and redistribution affect different margins. More capacity moderates housing-price increases, while a transfer changes the household's claim on the resulting resources. Their fiscal interaction matters: policies that reduce scarcity rents can also reduce the revenues of a programme financed exclusively from those rents. Maintaining a fixed transfer as supply expands may therefore require a changing rate or a broader funding base.

The incidence analysis also rejects an indiscriminate claim that cash support must be neutralized by landlords. The model specifies who finances the payment, who receives it, and how their marginal spending differs. Those details determine the price effect. The incomplete price offset proven here is conditional on competitive housing supply, the stated preferences, and balanced financing within the model. It should not be generalized to a deficit-financed transfer, a narrow housing voucher, a rent subsidy paid to suppliers, or a market with rationing and search frictions.

Testable restrictions and identification

A first empirical restriction concerns the gap between income growth and living-cost growth. For renter groups with similar expenditure preferences, markets with less responsive housing supply should exhibit a larger housing-price response to comparable positive income or output shocks, conditional on migration and housing-demand shifts. Evidence of low price responses in markets with demonstrably fixed supply would weaken this mechanism unless the measured shock changed housing preferences or left local purchasing power largely unchanged.

A second restriction concerns the interaction between income exposure and tenure. Households with weak claims on the nonhousing dividend and no housing-rent income should be more vulnerable to the same local price increase. Linked household income, asset, tenure, and expenditure records are needed to test that interaction. A renter-owner comparison alone is insufficient because the groups differ in age, wealth, family structure, neighbourhood, and labour-market exposure.

A third restriction concerns transfers. A balanced transfer between groups with similar marginal housing propensities should produce little aggregate housing-price movement in the common homothetic model. Price effects should be larger when recipients' housing propensities exceed those of funders and supply is less elastic. A finding of full recipient-benefit erosion would reject Proposition 4's maintained structure, rather than confirm the theory merely because housing prices increased.

A possible design would combine predetermined occupational exposure to AI with pre-existing housing-supply constraints and household tenure. Outcomes would include employment, full disposable income, quality-adjusted rents, housing quantities, and group-specific expenditure indices. Within-market household comparisons could separate income exposure from a common rent change. Across-market comparisons would additionally require location-specific trends and tests for differential migration, remote work, construction, and amenity changes. Exposure is not adoption, and adoption is not exogenous. A shift-share measure or event-study pattern would require an explicit identification argument rather than an automatic causal interpretation.

The substitution extension adds a measurable restriction linking the observed housing budget share, the price response, and real-income changes. It would be preferable to estimate these components separately and test the model's implied welfare response out of sample. None of these empirical exercises is reported as completed in this paper.

LIMITATIONS

The model takes the output dividend and income-share schedule as given. It does not solve firms' AI adoption, wages, hours, saving, capital accumulation, or the technology sector's pricing. Its equilibrium is conditional on those incomes and capacities. As a result, it identifies how a specified dividend is transmitted through goods markets, rather than predicting the size or distribution of the dividend itself.

Housing is a homogeneous service, and consumption substitution is summarized by restrictive functional forms. There are no mortgages, capital gains, vacancies, rent controls, tenure transitions, public housing allocation rules, or neighbourhood amenities. The tax applies to measured net rents with perfect administration and no avoidance. These assumptions are particularly consequential for implementation and should be relaxed before converting an illustrative rate into a policy recommendation.

The baseline separates owners and renters sharply and uses fixed populations. Migration could redistribute both people and demand; construction can affect wages and costs; AI may also lower construction costs or change the value of location. Any of those responses could weaken or strengthen the channel. The capacity parameter and supply elasticity should therefore be interpreted as conditional summaries, not technological constants.

Finally, numerical verification establishes that the stated equations generate the reported results. It does not validate the equations against an economy or establish the historical originality of every mathematical step. The new contribution claimed here is the combined incidence and compensation framework, bounded by the related literature discussed in Section 2.

CONCLUSION

Whether households benefit from AI depends on both the incomes they receive and the prices they face. In the benchmark model, nominal renter income can increase while consumption-equivalent welfare falls when income-share erosion lies between the housing-adjusted welfare threshold and one. Elastic housing supply narrows that region. Limited consumption substitution can widen it substantially.

A transfer financed from net housing rents can compensate renters under explicit conditions, but its required rate must be checked against the available rent base. Balanced redistribution is price-neutral under common homothetic preferences and partly offset by higher prices when recipients have a greater propensity to spend on housing. The resulting agenda is empirical: measure the distribution of the productivity dividend together with housing supply, ownership, and substitution, and evaluate household welfare rather than infer it from output growth alone.

RESEARCH TRANSPARENCY

Data and code availability. All numerical results are generated from the equations and disclosed scenario grids. No confidential or participant-level data are used. The accompanying replication archive contains the Python source, machine-readable tables, parameter-grid results, figure files, and numerical audit output. No public repository deposit is claimed.

Ethics. This theoretical and computational study involves no human participants, personal records, or interventions.

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APPENDIX A PROOFS AND ACCOUNTING CHECKS

A1 Benchmark equilibrium and resource feasibility

Summing household demands gives $R = \theta(Y + \Pi)$. Substituting $\Pi = \frac{R}{1+\eta}$ and rearranging yields Equation (8), whose denominators are strictly positive. Because $R = \kappa p^{1+\eta}$ is strictly increasing in p, the positive price is unique. The total nonhousing use is $(1-\theta)(Y+\Pi)+\eta\Pi$. Equation (8) implies $(1+\eta-\theta)\Pi = \theta Y$, so total nonhousing use equals Y. Both groups' incomes remain positive under the stated domains. The argument also holds when supply is fixed and $C = 0$.

A2 Welfare and compensation

At zero transfer, $\frac{I_r(g)}{I_r(1)} = g^{1-\gamma}$ and $\frac{p_g}{p_0} = g^{\frac{1}{1+\eta}}$. Substitution into Equation (6) gives Equation (9). Comparing its exponent to zero proves Proposition 2; comparing the income exponent to zero establishes Equation (10). Solving the same inequality for η gives Equation (11). Housing demand has elasticity $1-\gamma-\frac{1}{1+\eta} = \frac{\eta}{1+\eta} - \gamma$.

At a positive tax rate, renter welfare relative to baseline is $\frac{g^{\varepsilon}}{s_0}\left(s(g) + \frac{\tau\theta}{1+\eta-\theta}\right)$. This is strictly increasing in τ. Its unique value equal to one gives Equation (12), with a zero floor when renters already gain. Multiplying that rate by $\frac{\Pi}{Y}$ yields Equation (13). This derivation proves necessity as well as sufficiency within the model: a smaller rate cannot compensate a losing renter group.

A3 Transfers with heterogeneous preferences

Housing demand equals $\theta_r(sY+T)+\theta_o[(1-s)Y+\Pi-T]$. Collecting terms gives $R=\omega Y+\Delta T+\theta_o\Pi$. Substituting Equation (3) proves Equation (14). Imposing $T=\tau\Pi$ then proves Equation (15). Summing nonhousing expenditures and resource cost again gives Y, so the price feedback does not introduce an unfinanced source of demand.

For a general feasible balanced transfer, differentiation gives $\frac{d\ln p}{dT}=\frac{\Delta}{(1+\eta)(\omega Y+\Delta T)}$. Differentiate $V_r=\frac{sY+T}{p^{\theta_r}}$ and multiply by p^{θ_r} to obtain Equation (16). The fraction involving Δ is between zero and one because $\theta_o Y>0$, establishing Equation (17). Finally, Equation (15) implies $\frac{dT}{d\tau}=\frac{\omega Y(1+\eta-\theta_o)}{(1+\eta-\theta_o-\Delta\tau)^2}>0$. Therefore the recipient gain also holds along the tax-financed transfer schedule.

A4 CES equilibrium and welfare derivative

Common CES demand implies $R=b(p)(Y+\Pi)$. Using Equation (3), one obtains $Y=R\left(\frac{1}{b(p)}-\frac{1}{1+\eta}\right)$. Inserting $R=\kappa p^{1+\eta}$ and the housing share from Equation (18) gives Equation (19). Both powers on its right-hand side are positive, and the first coefficient is strictly positive. Thus the right-hand side is continuous and strictly increasing from zero to infinity, proving existence and uniqueness.

Logarithmic differentiation uses $\frac{d\ln b}{d\ln p}=(1-\sigma)(1-b)$. Solving for $\frac{d\ln p}{d\ln Y}$ produces the first part of Equation (20). The expenditure function satisfies $\frac{d\ln P}{d\ln p}=b$; applying this identity to $V_r=\frac{sY}{P}$ gives the second part. Under fixed supply, Equation (19) reduces to $Y=\frac{\kappa(1-\theta)p^{\sigma}}{\theta}$, which directly confirms $\varepsilon_p=\frac{1}{\sigma}$. Equation (21) follows from the expenditure required to attain baseline utility at the new price.

APPENDIX B REPRODUCIBILITY SPECIFICATION

The 5,880-case benchmark grid is the Cartesian product of housing shares {0.15, 0.25, 0.35, 0.45, 0.55}; supply elasticities {0, 0.25, 0.5, 1, 2, 4}; erosion elasticities {0, 0.2, 0.4, 0.6, 0.8, 1, 1.2}; output multipliers {1.01, 1.05, 1.10, 1.25, 1.50, 2, 3}; and initial renter shares {0.30, 0.50, 0.65, 0.80}. Closed-form market clearing, the welfare identity, and feasible compensation are checked in every case.

The 1,440-case CES grid combines housing weights {0.15, 0.35, 0.55}; supply elasticities {0, 0.25, 1, 4}; substitution elasticities {0.25, 0.5, 0.75, 1, 1.5, 2}; output multipliers {1.01, 1.10, 1.50, 2, 3}; and erosion elasticities {0, 0.4, 0.8, 1.2}, holding $s_0=0.65$. A bracketed Brent solver evaluates Equation (19) with absolute price tolerance 10^{-13}. Centred finite differences use a step of 10^{-5} in log output. Unit-substitution cases must agree with the closed-form Cobb-Douglas solution.

The 324-case heterogeneous-preference grid combines renter housing propensities {0.25, 0.45, 0.65}; owner propensities {0.10, 0.20, 0.25}; supply elasticities {0, 0.5, 2}; renter output shares {0.20, 0.45, 0.70}; and net-rent tax rates {0, 0.10, 0.50, 0.90}. Output equals 1.5 and capacity equals one. Both market identities, the marginal-retention bound, and welfare monotonicity in the transfer rate are checked.

The replication script uses Python with NumPy, SciPy, and Matplotlib. All computations are deterministic and require no random seed. The archive records package versions, complete parameter outputs for the benchmark and CES grids, the heterogeneous scenario table, and the numerical tolerances. Running python model.py --out results regenerates the computational outputs. The manuscript's displayed tables are rounded; the CSV files preserve unrounded values.