



Nano Semi C^* Generalized Separation Axioms in Nano Topological Spaces

Raja Mohammad Latif

Department of Mathematics and Natural Sciences Analysis of Prince Mohammad Bin Fahd University
P.O. Box 1664 Al Khobar 31952 Kingdom of Saudi Arabia Kingdom of Saudi Arabia

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ABSTRACT

Lellis Thivagar introduced the notion of Nano topology by using theory approximations and boundary region of a subset of a universe in terms of an equivalence relation on it and also defined nano closed sets, nano-interior and nano-closure in a nano topological space. In 2019, A. Pushpalatha and S. Sridevi introduced and investigated a new class of nano closed sets called nano semi c^* generalized closed sets. They studied its basic properties and its relation to various other nano closed sets. The class of sets namely N_{ano} -open sets are playing more important role in topological spaces, because of their applications in various fields of Mathematics and other real fields. This paper is devoted to the introduction of N_{ano} - sc^*g -separation Axioms namely N_{ano} - sc^*g - T_0 space, N_{ano} - sc^*g - T_1 space, N_{ano} - sc^*g - T_2 space, N_{ano} - sc^*g -regular space, N_{ano} - sc^*g -normal space, almost N_{ano} - sc^*g -normal space, mildly N_{ano} - sc^*g -normal space, N_{ano} - sc^*g -compact space, N_{ano} - sc^*g -connected space, N_{ano} - sc^*g -closed graph and strongly N_{ano} - sc^*g -closed graph in Nano topological spaces. We investigate the fundamental properties and characterizations of these spaces in Nano topological spaces.

IndexTerms— N_{ano} - sc^*g - T_1 -space, N_{ano} - sc^*g -regular space, N_{ano} - sc^*g -normal space, N_{ano} - sc^*g -compact space, N_{ano} - sc^*g -connected space, N_{ano} - sc^*g -closed graph

INTRODUCTION

Topology and its branches have become hot topics not only for almost all fields of mathematics, but also for many areas of science such as chemistry, and information systems. General Topology is vast and has many different inventions and interactions with other fields of Mathematics and Science. Lellis Thivagar introduced the notion of Nano topology by using theory approximations and boundary region of a subset of a universe in terms of an equivalence relation on it and defined Nano open sets, Nano closed sets, Nano-interior, and Nano-closure in Nano topological spaces. This paper is devoted to the introduction of N_{ano} - sc^*g -separation Axioms namely N_{ano} - sc^*g - T_0 space, N_{ano} - sc^*g - T_1 space, N_{ano} - sc^*g - T_2 space, N_{ano} - sc^*g -regular space, N_{ano} - sc^*g -normal space, almost N_{ano} - sc^*g -normal space, mildly N_{ano} - sc^*g -normal space, N_{ano} - sc^*g -compact space, N_{ano} - sc^*g -connected space, N_{ano} - sc^*g -closed graph and strongly N_{ano} - sc^*g -closed graph in Nano topological spaces. We investigate the fundamental properties and characterizations of these spaces in Nano topological space.

PRELIMINARIES

Definition 2.1. Let U be the universe, R be an equivalence relation on U and $T_R(X) = \{U \setminus \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $T_R(X)$ satisfies the following axioms.

- (1) U and $\phi \in T_R(X)$.



(2) The union of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

(3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X , $(U, \tau_R(X))$ is called the Nano topological space (Briefly N_{ano} -Top-Space). The elements of $\tau_R(X)$ are called as nano-open sets. The complements of the nano-open sets are called nano-closed sets.

Definition 2.2. Let $(U, \tau_R(X))$ be a N_{ano} -Top-Space and $A \subseteq X$. Then

*. The Nano interior of a set A is defined as the union of all N_{ano} -open subsets contained in A and it is denoted by $N_{ano}-Int(A)$. $N_{ano}-Int(A)$ is the largest N_{ano} -open subset of A .

**. The Nano closure of a set A is defined as the intersection of all N_{ano} -closed sets containing A and it is denoted by $N_{ano}-Cl(A)$. $N_{ano}-Cl(A)$ is the smallest N_{ano} -closed set containing A .

Definition 2.3. Let $(U, \tau_R(X))$ be a N_{ano} -Top-Space and $A \subseteq U$. Then A is called a

(1) N_{ano} -pre-closed set if $N_{ano}-Cl[N_{ano}-Int(A)] \subseteq A$.

(2) N_{ano} -semi-closed set if $N_{ano}-Int[N_{ano}-Cl(A)] \subseteq A$.

(3) N_{ano} - α closed set if $N_{ano}-Cl[N_{ano}-Int(N_{ano}-Cl(A))]$ $\subseteq A$.

(4) N_{ano} -regular-open set if $A = N_{ano}-Int[N_{ano}-Cl(A)]$.

(5) N_{ano} -regular-closed set if $N_{ano}-Cl[N_{ano}-Int(A)] = A$.

(6) N_{ano} -generalized closed (briefly $N_{ano}g$ -closed) set if $N_{ano}-Cl(A) \subseteq G$, whenever $A \subseteq G$ and G is N_{ano} -open set in U .

(7) N_{ano} -s-closed set if $N_{ano}-Cl[N_{ano}-Int(A)] \subseteq A$.

(8) N_{ano} -generalized semi closed (briefly $N_{ano}gs$ -closed) set if $N_{ano}s-Cl(A) \subseteq G$ whenever $A \subseteq G$ and G is N_{ano} -open set in U .

(9) N_{ano} - β -closed set if $N_{ano}-Cl[N_{ano}-Int(N_{ano}-Cl(A))]$ $\subseteq A$.

(8) N_{ano} - α -open set if $A \subseteq N_{ano}-Int[N_{ano}-Cl(N_{ano}-Int(A))]$.

(10) Nano c^* -set (briefly $N_{ano}-c^*$ -set) if $A = G \cap F$, where G is N_{ano} - g -open set and F is N_{ano} - α -open set in U .

(11) Nano α generalized closed (briefly $N_{ano}\alpha g$ -closed) set if $N_{ano}\alpha-Cl(A) \subseteq G$, whenever $A \subseteq G$ and G is N_{ano} -open in U .

The complements of the above nano open (closed) sets are called their nano closed (resp. open) sets.

Definition 2.4. A subset A of a N_{ano} -Top-Space $(U, T_R(X))$ is called a nano semi c^* generalized closed (briefly N_{ano} -sc $\star$ g-closed) set if N_{ano} -sCl(A) $\subseteq G$, whenever $A \subseteq G$ and G is N_{ano} -c $\star$ -set in $(U, T_R(X))$.

Definition 2.5. Let $(U, T_R(X))$ be a N_{ano} -Top-Space and $A \subseteq U$. Then A is called N_{ano} -sc $\star$ g-open, if A^c is N_{ano} -sc $\star$ g-closed. The family of all N_{ano} -sc $\star$ g-open (resp. N_{ano} -sc $\star$ g-closed) sets in $(U, T_R(X))$ is denoted by N_{ano} -sc $\star$ g-O(U) = N_{ano} -sc $\star$ g-O($U, T_R(X)$) (resp. N_{ano} -sc $\star$ g-C(U) = N_{ano} -sc $\star$ g-C($U, T_R(X)$)).

Definition 2.6. Let $(U, T_R(X))$ be a N_{ano} -Top-Space and $A \subseteq U$. Then the N_{ano} -sc $\star$ g-interior and N_{ano} -sc $\star$ g-closure of A are defined and denoted as follows:
 N_{ano} -sc $\star$ g-Int(A) = $\bigcup \{G : G \in N_{ano}$ -sc $\star$ g-O($U, T_R(X)$) and $G \subseteq A\}$.

N_{ano} -sc $\star$ g-Cl(A) = $\bigcap \{H : H \in N_{ano}$ -sc $\star$ g-C($U, T_R(X)$) and $A \subseteq H\}$.

Theorem 2.7. A N_{ano} -closed set A in a N_{ano} -Top-Space $(U, T_R(X))$ is N_{ano} -sc $\star$ g-closed set.

Proof. Let A be a N_{ano} -closed set in U . Let G be a N_{ano} -c $\star$ -set such that $A \subseteq G$. Since A is N_{ano} -closed set we have N_{ano} -Cl(A) = $A \subseteq G$. But, N_{ano} -sCl(A) $\subseteq N_{ano}$ -Cl(A) $\subseteq G$. Therefore N_{ano} -sCl(A) $\subseteq G$ where G is N_{ano} -c $\star$ -set. Hence A is a N_{ano} -sc $\star$ g-closed set in U .

Theorem 2.8. In a N_{ano} -Top-Space $(U, T_R(X))$, the intersection of any two N_{ano} -sc $\star$ g-closed sets is N_{ano} -sc $\star$ g-closed.

Theorem 2.9. Let A and B be N_{ano} -sc $\star$ g-open sets in a N_{ano} -Top-Space $(U, T_R(X))$. Then $A \cup B$ is also N_{ano} -sc $\star$ g-open set in $(U, T_R(X))$.

Proof. Let A and B be N_{ano} -sc $\star$ g-open sets in $(U, T_R(X))$. Then A^c and B^c are N_{ano} -sc $\star$ g-closed sets in $(U, T_R(X))$. By Theorem 2.8, $A^c \cap B^c = (A \cup B)^c$ is a N_{ano} -sc $\star$ g-closed set in $(U, T_R(X))$. Therefore $A \cap B$ is N_{ano} -sc $\star$ g-open set in $(U, T_R(X))$.

Definition 2.10. A function $f : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is called N_{ano} -sc $\star$ g-irresolute if $f^{-1}(H)$ is N_{ano} -sc $\star$ g-open set in $(U, T_R(X))$ for each N_{ano} -sc $\star$ g-open set H in $(V, T_{R'}(Y))$.

Definition 2.11. A function $f : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is called quasi N_{ano} -sc $\star$ g-irresolute if for each $x \in U$ and for each $H \in N_{ano}$ -sc $\star$ g-O($V, T_{R'}(Y)$) with $f(x) \in H$, there exist $G \in N_{ano}$ -sc $\star$ g-O($U, T_R(X)$) such that $x \in G$ and $f(G) \subseteq N_{ano}$ -sc $\star$ g-Cl(H).

Definition 2.12. A function $f : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is called N_{ano} -sc $\star$ g-continuous if $f^{-1}(H)$ is N_{ano} -sc $\star$ g-open set in $(U, T_R(X))$ for each N_{ano} -open set H in $(V, T_{R'}(Y))$.

Definition 2.13. A function $f : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is called N_{ano} -sc $\star$ g-closed if the image of every N_{ano} -closed set in $(U, T_R(X))$ is N_{ano} -sc $\star$ g-closed in $(V, T_{R'}(Y))$.

Definition 2.14. A function $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is said to be N_{ano} - $\neg sc \star g$ -open if $f(G)$ is N_{ano} - $\neg sc \star g$ -open for each N_{ano} -open set G in $(U, T_R(X))$.

Definition 2.15. A function $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is said to be strongly N_{ano} - $\neg sc \star g$ -closed if the image $f(G)$ is N_{ano} - $\neg sc \star g$ -closed in $(V, T_{R'}(Y))$ for each N_{ano} - $\neg sc \star g$ -closed G in $(U, T_R(X))$.

Definition 2.16. A function $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is said to be strongly N_{ano} - $\neg sc \star g$ -open if the image $f(G)$ is N_{ano} - $\neg sc \star g$ -open set in $(V, T_{R'}(Y))$ for each N_{ano} - $\neg sc \star g$ -open set G in $(U, T_R(X))$.

Definition 2.17. A function $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is said to be contra N_{ano} - $\neg sc \star g$ -closed (resp. contra N_{ano} - $\neg sc \star g$ -open) if the image of every N_{ano} -closed (resp. N_{ano} -open) set in $(U, T_R(X))$ is N_{ano} - $\neg sc \star g$ -open (resp. N_{ano} - $\neg sc \star g$ -closed) set in $(V, T_{R'}(Y))$.

3. Nano Semi C* Generalized Separation Axioms

In this section, we introduce and study the concepts of N_{ano} - $\neg sc \star g$ - T_0 -Space, N_{ano} - $\neg sc \star g$ - T_1 -Space, and N_{ano} - $\neg sc \star g$ - T_2 -Space via N_{ano} - $\neg sc \star g$ -open sets in N_{ano} -Top-Spaces.

Definition 3.1. A N_{ano} -Top-Space $(U, T_R(X))$ is called N_{ano} - T_0 -Space, if for any two points $x, y \in U$ such that $x \neq y$, there exists a N_{ano} -open set containing one of them but not the other.

Definition 3.2. A N_{ano} -Top-Space $(U, T_R(X))$ is called N_{ano} - $\neg sc \star g$ - T_0 -Space, if for any two points $x, y \in U$ with $x \neq y$, there exists a N_{ano} - $\neg sc \star g$ -open set containing one of them but not the other.

Theorem 3.3. A N_{ano} -Top-Space $(U, T_R(X))$ is N_{ano} - $\neg sc \star g$ - T_0 -Space, if and only if N_{ano} - $\neg sc \star g$ -closures of distinct points are distinct.

Proof. Let x, y be distinct points of U . Since U is a N_{ano} - $\neg sc \star g$ - T_0 -Space, there exists a N_{ano} - $\neg sc \star g$ -open set G such that $x \in G$ and $y \notin G$. Consequently, $U - G$ is a N_{ano} - $\neg sc \star g$ -closed set containing y but not x . But $N_{ano}\neg sc \star g\text{-Cl}(\{y\})$ is the intersection of all N_{ano} - $\neg sc \star g$ -closed sets containing y . Hence $y \in N_{ano}\neg sc \star g\text{-Cl}(\{y\})$ but $x \notin N_{ano}\neg sc \star g\text{-Cl}(\{y\})$ as $x \notin U - G$. Therefore $N_{ano}\neg sc \star g\text{-Cl}(\{x\}) \neq N_{ano}\neg sc \star g\text{-Cl}(\{y\})$.

Conversely, let $N_{ano}\neg sc \star g\text{-Cl}(\{x\}) \neq N_{ano}\neg sc \star g\text{-Cl}(\{y\})$ for $x \neq y$. Then there exists at least one point $z \in U$ such that $z \in N_{ano}\neg sc \star g\text{-Cl}(\{x\})$ and $z \notin N_{ano}\neg sc \star g\text{-Cl}(\{y\})$. Suppose $x \notin N_{ano}\neg sc \star g\text{-Cl}(\{y\})$ because if $x \in N_{ano}\neg sc \star g\text{-Cl}(\{y\})$, then $\{x\} \subseteq N_{ano}\neg sc \star g\text{-Cl}(\{y\})$ implies $N_{ano}\neg sc \star g\text{-Cl}(\{x\}) \subseteq N_{ano}\neg sc \star g\text{-Cl}(\{y\})$. So $z \in N_{ano}\neg sc \star g\text{-Cl}(\{y\})$ which is a contradiction. Hence $x \notin N_{ano}\neg sc \star g\text{-Cl}(\{y\})$ which implies $x \in H = [U - [N_{ano}\neg sc \star g\text{-Cl}(\{y\})]]$ which is a N_{ano} - $\neg sc \star g$ -open set containing x but not y . Hence U is a N_{ano} - $\neg sc \star g$ - T_0 -Space.

Theorem 3.4. If $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is an injective, N_{ano} - $\neg sc \star g$ -irresolute function and $(V, T_{R'}(Y))$ is a N_{ano} - $\neg sc \star g$ - T_0 -Space, then $(U, T_R(X))$ is a N_{ano} - $\neg sc \star g$ - T_0 -Space.

Proof. Suppose $\mathcal{V}$ is a $N_{ano}\text{-}sc^*g\text{-}T_0$ -Space. Let x and y be any two distinct points in $\mathcal{U}$. Since f is injective, so $f(x)$ and $f(y)$ are distinct points in $\mathcal{V}$. Since $\mathcal{V}$ is a $N_{ano}\text{-}sc^*g\text{-}T_0$ -Space, there exists a $N_{ano}\text{-}sc^*g$ -open set H in $\mathcal{V}$ containing $f(x)$ but not $f(y)$. Again since f is $N_{ano}\text{-}sc^*g$ -irresolute, $f^{-1}(H)$ a $N_{ano}\text{-}sc^*g$ -open set in $\mathcal{U}$ containing x but not y . Therefore $\mathcal{U}$ is a $N_{ano}\text{-}sc^*g\text{-}T_0$ -Space.

Theorem 3.5. If $f:(\mathcal{U}, T_R(X)) \rightarrow (\mathcal{V}, T_{R'}(Y))$ is an injective, $N_{ano}\text{-}sc^*g$ -continuous function and $(\mathcal{V}, T_{R'}(Y))$ is a $N_{ano}\text{-}T_0$ -Space, then $(\mathcal{U}, T_R(X))$ is a $N_{ano}\text{-}sc^*g\text{-}T_0$ -Space.

Proof. Suppose $(\mathcal{V}, T_{R'}(Y))$ is a $N_{ano}\text{-}sc^*g\text{-}T_0$ -Space. Let x and y be any two distinct points in $\mathcal{U}$. Since f is injective, so $p = f(x)$ and $q = f(y)$ are distinct point in $\mathcal{V}$. Since $(\mathcal{V}, T_{R'}(Y))$ is a $N_{ano}\text{-}T_0$ -Space, there exists a N_{ano} -open set H in $\mathcal{V}$ containing $p = f(x)$ but not $q = f(y)$. Again since f is $N_{ano}\text{-}sc^*g$ -continuous, $G = f^{-1}(H)$ is a $N_{ano}\text{-}sc^*g$ -open set in $\mathcal{U}$ containing x but not y . Therefore $(\mathcal{U}, T_R(X))$ is a $N_{ano}\text{-}sc^*g\text{-}T_0$ -Space.

Definition 3.6. A N_{ano} -Top-Space $(\mathcal{U}, T_R(X))$ is called $N_{ano}\text{-}T_1$ -Space, if for any two points $x \neq y$, there exist two N_{ano} -open sets G and H with $x \in G$, $y \notin G$ and $x \notin H$, $y \in H$.

Definition 3.7. A N_{ano} -Top-Space $(\mathcal{U}, T_R(X))$ is called $N_{ano}\text{-}sc^*g\text{-}T_1$ -Space, if for any two points $x \neq y$, there exist two $N_{ano}\text{-}sc^*g$ -open sets G and H with $x \in G$, $y \notin G$ and $x \notin H$, $y \in H$.

Theorem 3.8. A N_{ano} -Top-Space $(\mathcal{U}, T_R(X))$ is $N_{ano}\text{-}sc^*g\text{-}T_1$ -Space if every singleton subset $\{x\}$ of $\mathcal{U}$ is a $N_{ano}\text{-}sc^*g$ -closed set in $(\mathcal{U}, T_R(X))$.

Proof. Suppose $\{x\}$ is a $N_{ano}\text{-}sc^*g$ -closed set for every $x \in \mathcal{U}$. Let x and $y \in \mathcal{U}$ with $x \neq y$. Now $x \neq y$ implies $y \in \mathcal{U} - \{x\}$. Hence $\mathcal{U} - \{x\}$ is a $N_{ano}\text{-}sc^*g$ -open set containing y but not x . Similarly $\mathcal{U} - \{y\}$ is a $N_{ano}\text{-}sc^*g$ -open set containing x but not y . Hence $\mathcal{U}$ is a $N_{ano}\text{-}sc^*g\text{-}T_1$ -Space.

Theorem 3.9. If $f:(\mathcal{U}, T_R(X)) \rightarrow (\mathcal{V}, T_{R'}(Y))$ is an injective, $N_{ano}\text{-}sc^*g$ -irresolute function and $(\mathcal{V}, T_{R'}(Y))$ is a $N_{ano}\text{-}sc^*g\text{-}T_1$ -Space, then $(\mathcal{U}, T_R(X))$ is a $N_{ano}\text{-}sc^*g\text{-}T_1$ -Space.

Proof. Let p and q be pair of distinct points in $\mathcal{U}$. Since f is injective, there exist two distinct points x and y of $\mathcal{V}$ such that $f(p) = x$ and $f(q) = y$. Since $\mathcal{V}$ is a $N_{ano}\text{-}sc^*g\text{-}T_1$ -Space, there exist $N_{ano}\text{-}sc^*g$ -open sets G and H in $\mathcal{V}$ such that $x \in G$, $y \notin G$ and $x \notin H$, $y \in H$. That is $p \in f^{-1}(G)$, $p \notin f^{-1}(H)$ and $q \in f^{-1}(H)$, $q \notin f^{-1}(G)$. Since f is $N_{ano}\text{-}sc^*g$ -irresolute function, $P = f^{-1}(G)$, $Q = f^{-1}(H)$ are $N_{ano}\text{-}sc^*g$ -open sets in $\mathcal{U}$. Thus for two distinct points p and q of $\mathcal{U}$ there exist $N_{ano}\text{-}sc^*g$ -open sets P and Q such that $p \in P$, $p \notin Q$ and $q \in Q$, $q \notin P$. Therefore $(\mathcal{U}, T_R(X))$ is a $N_{ano}\text{-}sc^*g\text{-}T_1$ -Space.

Theorem 3.10. If $f:(\mathcal{U}, T_R(X)) \rightarrow (\mathcal{V}, T_{R'}(Y))$ is $N_{ano}\text{-}sc^*g$ -continuous, injection and $(\mathcal{V}, T_{R'}(Y))$ is a $N_{ano}\text{-}T_1$ -Space, then $(\mathcal{U}, T_R(X))$ is a $N_{ano}\text{-}sc^*g\text{-}T_1$ -Space.

Proof. Let p and q be two distinct points in $\mathcal{U}$. Then there exist two distinct points x and y of $\mathcal{V}$ such that $f(p) = x$ and $f(q) = y$. Since $\mathcal{V}$ is a $N_{ano}\text{-}T_1$ -Space, there exist N_{ano} -open sets G and H in $\mathcal{V}$ such that $x \in G$, $y \notin G$ and $x \notin H$, $y \in H$. That is $p \in f^{-1}(G)$, $p \notin f^{-1}(H)$ and $q \in f^{-1}(H)$, $q \notin f^{-1}(G)$. Since f is a $N_{ano}\text{-}sc^*g$ -continuous function, $P = f^{-1}(G)$, $Q = f^{-1}(H)$ are $N_{ano}\text{-}sc^*g$ -open sets in $\mathcal{U}$. Thus for two distinct

points p and q of U there exist N_{ano} - sc^*g -open sets P and Q such that $p \in P$ and $p \notin Q$ and $q \in Q$ and $q \notin P$. Therefore $(U, T_R(X))$ is a N_{ano} - sc^*g - T_1 -Space.

Definition 3.11. A N_{ano} -Top-Space $(U, T_R(X))$ is called N_{ano} - T_2 -Space, if for any two points $x \neq y$, there exists disjoint N_{ano} -open sets G and H with $x \in G$ and $y \in H$.

Definition 3.12. A N_{ano} -Top-Space $(U, T_R(X))$ is called N_{ano} - sc^*g - T_2 -Space, if for any two points $x \neq y$, there exists disjoint N_{ano} - sc^*g -open sets G and H with $x \in G$ and $y \in H$.

Theorem 3.13. If $f: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is a N_{ano} - sc^*g -continuous injection, and $(V, T_{R'}(Y))$ is a N_{ano} - T_2 -Space, then $(U, T_R(X))$ is a N_{ano} - sc^*g - T_2 -Space.

Proof. Let p and q be two distinct points in U . Then there exist two distinct points x and y of V such that $f(p) = x$ and $f(q) = y$. Since V is a N_{ano} - T_2 -Space, there exist disjoint N_{ano} -open sets G and H in V such that $x \in G$ and $y \in H$. That is $p \in f^{-1}(G)$ and $q \in f^{-1}(H)$. Since f is a N_{ano} - sc^*g -continuous function, $P = f^{-1}(G)$, $Q = f^{-1}(H)$ are N_{ano} - sc^*g -open sets in U . Further as f is injective, $P \cap Q = f^{-1}(G) \cap f^{-1}(H) = f^{-1}(\phi) = \phi$. Thus for any two distinct points p and q of U there exist disjoint N_{ano} - sc^*g -open sets P and Q such that $p \in P$ and $q \in Q$. Therefore $(U, T_R(X))$ is a N_{ano} - sc^*g - T_2 -Space.

Theorem 3.14. If $f: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is an injective, N_{ano} - sc^*g -irresolute function and $(V, T_{R'}(Y))$ is a N_{ano} - sc^*g - T_2 -Space, then $(U, T_R(X))$ is also a N_{ano} - sc^*g - T_2 -Space.

Proof. Let p and q be any two distinct points in U . Since f is injective, there exist two distinct points x and y of V such that $f(p) = x$ and $f(q) = y$. Since V is a N_{ano} - sc^*g - T_2 -Space, there exist disjoint N_{ano} - sc^*g -open sets G and H in V such that $x \in G$ and $y \in H$. That is $p \in f^{-1}(G)$ and $q \in f^{-1}(H)$. Since f is a N_{ano} - sc^*g -irresolute function, $P = f^{-1}(G)$, $Q = f^{-1}(H)$ are disjoint N_{ano} - sc^*g -open sets in U . Thus for any two distinct points p and q of U there exist disjoint N_{ano} - sc^*g -open sets P and Q such that $p \in P$ and $q \in Q$. Therefore $(U, T_R(X))$ is a N_{ano} - sc^*g - T_2 -Space.

Theorem 3.15. A N_{ano} -Top-Space $(U, T_R(X))$ is N_{ano} - sc^*g - T_2 -Space if and only if for each $x \neq y$ there exists a N_{ano} - sc^*g -open set G such that $x \in G$ and $y \notin N_{ano}$ - sc^*g -Cl(G).

Proof. Assume that U is a N_{ano} - sc^*g - T_2 -Space. Let $x, y \in U$ and $x \neq y$, then there exist disjoint N_{ano} - sc^*g -open sets G and H in U such that $x \in G$ and $y \in H$. Clearly, $U - H$ is N_{ano} - sc^*g -closed set. Since $G \cap H = \phi$, $G \subseteq U - H$. Therefore N_{ano} - sc^*g -Cl(G) $\subseteq$ N_{ano} - sc^*g -Cl($U - H$) = $U - H$. Now $y \notin U - H$ implies $y \notin N_{ano}$ - sc^*g -Cl(G).

Conversely, let $x, y \in U$ and $x \neq y$. By hypothesis, there exists a N_{ano} - sc^*g -open set G such that $x \in G$ and $y \notin N_{ano}$ - sc^*g -Cl(G). Let $H = U - [N_{ano}$ - sc^*g -Cl(G)]. Then G and H are two disjoint N_{ano} - sc^*g -open sets such that $x \in G$ and $y \in H$. Therefore $(U, T_R(X))$ is a N_{ano} - sc^*g - T_2 -Space.

Proposition 3.16. Every N_{ano} - T_k -Space $[k = 0, 1, 2]$ is N_{ano} - sc^*g - T_k -Space.



Theorem 3.17. If $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is a bijective strongly N_{ano} - sc^*g -open and U is N_{ano} - sc^*g - T_0 -Space, then V is also N_{ano} - sc^*g - T_0 -Space.

Proof. Let p and q be two distinct points of V . Since f is bijective there exist distinct points x and y of U such that $f(x)=p$ and $f(y)=q$. Since U is N_{ano} - sc^*g - T_0 -Space, there exists a N_{ano} - sc^*g -open set G such that $x \in G$ and $y \notin G$. Therefore $p = f(x) \in f(G)$ and $q = f(y) \notin f(G)$. Since f being strongly N_{ano} - sc^*g -open function, $f(G)$ is N_{ano} - sc^*g -open in V . Thus, there exists a N_{ano} - sc^*g -open set $f(G)$ in V such that $p \in f(G)$ and $q \notin f(G)$. Therefore V is N_{ano} - sc^*g - T_0 -Space.

Theorem 3.18. The property being N_{ano} - sc^*g - T_1 -Space is preserved under bijection and strongly N_{ano} - sc^*g -open function.

Proof. Let $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be bijective and strongly N_{ano} - sc^*g -open function. Let U be a N_{ano} - sc^*g - T_1 -Space and p, q be any two distinct points of V . Since f is bijective, there exist distinct points x, y of U such that $p = f(x)$ and $q = f(y)$. Now V being a N_{ano} - sc^*g - T_1 -Space, there exist N_{ano} - sc^*g -open sets G and H such that $x \in G, y \notin G$ and $x \notin H, y \in H$. Therefore $p = f(x) \in f(G)$ but $q = f(y) \notin f(G)$ and $p = f(x) \notin f(H)$. Now f being strongly N_{ano} - sc^*g -open, $f(G)$ and $f(H)$ are N_{ano} - sc^*g -open subsets of V such that $p \in f(G)$ but $q \notin f(G)$ and $q \in f(H)$ while $p \notin f(H)$. Hence V is N_{ano} - sc^*g - T_1 -Space.

4. Nano Semi C* Generalized Regular Spaces

In this section, we introduce and study the concepts of N_{ano} - sc^*g -regular and almost N_{ano} - sc^*g -regular spaces via N_{ano} - sc^*g -open sets in N_{ano} -Top-Spaces.

Definition 4.1. A N_{ano} -Top-Space $(U, T_R(X))$ is called N_{ano} -regular space, if for each N_{ano} -closed subset F of U and each point $x \in U - F$, there exist two disjoint N_{ano} -open sets G and H such that $x \in G$ and $F \subseteq H$.

Definition 4.2. A N_{ano} -Top-Space $(U, T_R(X))$ is called N_{ano} - sc^*g -regular space, if for any point $x \in U$ and N_{ano} -closed F of U such that $x \notin F$, there exist two disjoint N_{ano} - sc^*g -open sets G and H such that $x \in G$ and $F \subseteq H$.

Theorem 4.3. Every N_{ano} -regular space is N_{ano} - sc^*g -regular.

Theorem 4.4. If $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is N_{ano} - sc^*g -irresolute, N_{ano} -closed injection and V is N_{ano} - sc^*g -regular space then U is N_{ano} - sc^*g -regular space.

Proof. Let K be N_{ano} -closed set in U and $x \notin K$. Since f is injection, N_{ano} -closed $f(K)$ is N_{ano} -closed set in V such that $f(x) \notin f(K)$. Now V is N_{ano} - sc^*g -regular, there exist disjoint N_{ano} - sc^*g -open sets G and H such that $f(x) \in G$ and $f(K) \subseteq H$, which implies $x \in f^{-1}(G)$ and $K \subseteq f^{-1}(H)$. Since f is N_{ano} - sc^*g -irresolute, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint N_{ano} - sc^*g -open sets in U . Therefore U is N_{ano} - sc^*g -regular space.

Theorem 4.5. If $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is N_{ano} -continuous surjective, strongly N_{ano} - sc^*g -open function and U is N_{ano} - sc^*g -regular space then V is N_{ano} - sc^*g -regular space.



Proof. Let K be N_{ano} -closed set in V and $y \notin K$. Take $y = f(x)$ for some $x \in U$. Since f is N_{ano} -continuous surjective, $f^{-1}(K)$ is N_{ano} -closed set in U such that $x \notin f^{-1}(K)$. Now U is N_{ano} - sc^*g -regular, there exist disjoint N_{ano} - sc^*g -open sets G and H such that $x \in G$ and $f^{-1}(K) \subseteq H$, That is $y = f(x) \in f(G)$ and $K \subseteq f(H)$. Since f is strongly N_{ano} - sc^*g -open function, $f(G)$ and $f(H)$ are disjoint N_{ano} - sc^*g -open sets in V . Therefore V is N_{ano} - sc^*g -regular space.

Theorem 4.6. If $f : (U, T_R(X)) \rightarrow (V, T_R(Y))$ is N_{ano} - sc^*g -continuous, N_{ano} -closed injection and V is ultra-regular space then U is N_{ano} - sc^*g -regular space.

Proof. Let K be N_{ano} -closed set in U and $x \notin K$. Since f is N_{ano} -closed injection, $f(K)$ is N_{ano} -closed set in V such that $f(x) \notin f(K)$. Now V is ultra-regular, there exist disjoint N_{ano} -clopen sets G and H such that $f(x) \in G$ and $f(K) \subseteq H$, which implies $x \in f^{-1}(G)$ and $K \subseteq f^{-1}(H)$. Since f is N_{ano} - sc^*g -continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint N_{ano} - sc^*g -open sets in U . Therefore U is N_{ano} - sc^*g -regular space.

Theorem 4.7. If $f : (U, T_R(X)) \rightarrow (V, T_R(Y))$ is N_{ano} -continuous bijective, N_{ano} - sc^*g -open function and U is N_{ano} -regular space then V is N_{ano} - sc^*g -regular space.

Proof. Let K be N_{ano} -closed set in V and $y \notin K$. Take $y = f(x)$ for some $x \in U$. Since f is N_{ano} -continuous bijective, $f^{-1}(K)$ is N_{ano} -closed set in U such that $x \notin f^{-1}(K)$. Now U is N_{ano} -regular space, there exist disjoint open sets G and H such that $x \in G$ and $f^{-1}(K) \subseteq H$. That is $y = f(x) \in f(G)$ and $K \subseteq f(H)$. Since f is N_{ano} - sc^*g -open function, $f(G)$ and $f(H)$ are disjoint N_{ano} - sc^*g -open sets in V . Therefore V is N_{ano} - sc^*g -regular space.

Theorem 4.8. If $f : (U, T_R(X)) \rightarrow (V, T_R(Y))$ is N_{ano} -continuous bijective, contra N_{ano} - sc^*g -closed function and U is ultra-regular space then V is N_{ano} - sc^*g -regular space.

Proof. Let K be N_{ano} -closed set in V and $y \notin K$. Take $y = f(x)$ for some $x \in U$. Since f is N_{ano} -continuous bijective, $f^{-1}(K)$ is N_{ano} -closed set in U such that $x \notin f^{-1}(K)$. Now U is ultra-regular, there exist disjoint clopen sets G and H such that $x \in G$ and $f^{-1}(K) \subseteq H$. That is $y = f(x) \in f(G)$ and $K \subseteq f(H)$. Since f is contra N_{ano} - sc^*g -closed function, $f(G)$ and $f(H)$ are disjoint N_{ano} - sc^*g -open sets in V . Therefore V is N_{ano} - sc^*g -regular space.

Theorem 4.9. If $f : (U, T_R(X)) \rightarrow (V, T_R(Y))$ is N_{ano} - sc^*g -continuous, N_{ano} -closed injection and V is N_{ano} -regular space, then U is N_{ano} - sc^*g -regular space.

Proof. Let K be N_{ano} -closed set in U and $x \notin K$. Since f is injection, N_{ano} -closed $f(K)$ is N_{ano} -closed set in V such that $f(x) \notin f(K)$. Now V is N_{ano} -regular, there exist disjoint N_{ano} -open sets G and H such that $f(x) \in G$ and $f(K) \subseteq H$, which implies $x \in f^{-1}(G)$ and $K \subseteq f^{-1}(H)$. Since f is N_{ano} - sc^*g -continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint N_{ano} - sc^*g -open sets in U . Therefore U is N_{ano} - sc^*g -regular space.

Theorem 4.10. If $f : (U, T_R(X)) \rightarrow (V, T_R(Y))$ is N_{ano} -continuous bijective, N_{ano} - sc^*g -open function and U is ultra-regular space then V is N_{ano} - sc^*g -regular space.

Proof. Let K be N_{ano} -closed set in V and $y \notin K$. Take $y = f(x)$ for some $x \in U$. Since f is N_{ano} -continuous bijective, $f^{-1}(K)$ is N_{ano} -closed set in U such that $x \notin f^{-1}(K)$. Now U is ultra regular, there exist disjoint



clopen sets G and H such that $x \in G$ and $f^{-1}(K) \subseteq H$, That is $y = f(x) \in f(G)$ and $K \subseteq f(H)$. Since f is N_{ano} - sc^*g -open function, $f(G)$ and $f(H)$ are disjoint N_{ano} - sc^*g -open sets in V . Therefore V is N_{ano} - sc^*g -regular space.

Theorem 4.11. If $f : (U, T_R(X)) \rightarrow (V, T_R(Y))$ is N_{ano} -continuous surjective, strongly N_{ano} - sc^*g -open function and U is ultra-regular space then V is N_{ano} - sc^*g -regular space.

Proof. Let K be N_{ano} -closed set in V and $y \notin K$. Take $y = f(x)$ for some $x \in U$. Since f is N_{ano} -continuous surjective, $f^{-1}(K)$ is N_{ano} -closed set in U such that $x \notin f^{-1}(K)$. Now U is ultra-regular, there exist disjoint clopen sets G and H such that $x \in G$ and $f^{-1}(K) \subseteq H$, We know that every N_{ano} -open set is N_{ano} - sc^*g -open. That is $y = f(x) \in f(G)$ and $K \subseteq f(H)$. Since f is strongly N_{ano} - sc^*g -open function, $f(G)$ and $f(H)$ are disjoint N_{ano} - sc^*g -open sets in V . Therefore V is N_{ano} - sc^*g -regular space.

Theorem 4.12. Let $(U, T_R(X))$ be any N_{ano} -Top-Space. Then the following statements are equivalent:

- (i) U is N_{ano} - sc^*g -regular.
- (ii) For every point $x \in U$ and N_{ano} -open G containing x there exists N_{ano} - sc^*g -open set H such that $x \in H \subseteq N_{ano}$ - sc^*g - $Cl(H) \subseteq G$.
- (iii) For every N_{ano} -closed F , $F = \bigcap \{N_{ano}$ - sc^*g - $Cl(G) : F \subseteq G \text{ and } G \in N_{ano}$ - sc^*g - $O(U, T_R(X))\}$.
- (iv) For every set K and a N_{ano} -open set L such that $K \cap L \neq \emptyset$, there exists N_{ano} - sc^*g -open O such that $K \cap O \neq \emptyset$ and N_{ano} - sc^*g - $Cl(O) \subseteq L$.
- (v) For every nonempty set K and N_{ano} -closed L such that $K \cap L \neq \emptyset$, there exists disjoint N_{ano} - sc^*g -open sets S and P such that $K \cap S \neq \emptyset$ and $L \subseteq P$.

Proof. (i) $\Rightarrow$ (ii). Let G be a N_{ano} -open G set containing x . Then $U - G$ is N_{ano} -closed set not containing x . Since U is N_{ano} - sc^*g -regular, there exists N_{ano} - sc^*g -open sets S and H such that $x \in H$, $U - G \subseteq S$ and $H \cap S = \emptyset$. This implies $H \subseteq U - S$. Therefore N_{ano} - sc^*g - $Cl(H) \subseteq N_{ano}$ - sc^*g - $Cl(U - S) = U - S$. because $U - S$ is N_{ano} - sc^*g -closed. Hence it implies that $x \in H \subseteq N_{ano}$ - sc^*g - $Cl(H) \subseteq U - S \subseteq G$. That is $x \in H \subseteq N_{ano}$ - sc^*g - $Cl(H) \subseteq G$.

(ii) $\Rightarrow$ (iii). Let F be a N_{ano} -closed set and $x \notin F$. Then $U - F$ is a N_{ano} -open containing x . By (ii), there is a N_{ano} - sc^*g -open set H such that $x \in H \subseteq N_{ano}$ - sc^*g - $Cl(H) \subseteq U - F$. Hence, $F \subseteq U - [N_{ano}$ - sc^*g - $Cl(H)] \subseteq U - H$. Consequently, $U - H$ is N_{ano} - sc^*g -closed set not containing x . Put $G = U - [N_{ano}$ - sc^*g - $Cl(H)]$. This implies $F \subseteq G$ and G is N_{ano} - sc^*g -open set of U and $x \notin N_{ano}$ - sc^*g - $Cl(G)$, implies $\bigcap \{N_{ano}$ - sc^*g - $Cl(G) : F \subseteq G \text{ and } G \in N_{ano}$ - sc^*g - $O(U, T_R(X))\} \subseteq F$. But F is N_{ano} -closed and every N_{ano} -closed set is N_{ano} - sc^*g -closed. Therefore $F \subseteq \bigcap \{N_{ano}$ - sc^*g - $Cl(G) : F \subseteq G \text{ and } G \in N_{ano}$ - sc^*g - $O(U, T_R(X))\}$ is always true. Thus $F = \bigcap \{N_{ano}$ - sc^*g - $Cl(G) : F \subseteq G \text{ and } G \in N_{ano}$ - sc^*g - $O(U, T_R(X))\}$.

(iii) $\Rightarrow$ (iv). Let $K \cap L \neq \emptyset$ and L is N_{ano} -open. Let $x \in K \cap L$. Then $U - L$ is a N_{ano} -closed set not containing x . By (iii) there exists a N_{ano} - sc^*g -open set G of U such that $U - L \subseteq G$ and $x \notin N_{ano}$ - sc^*g - $Cl(G)$. Put



$O = U - [N_{ano} -sc *g -Cl(G)]$, then O is $N_{ano} -sc *g$ -open set of U , $x \in K \cap O$ and $N_{ano} -sc *g -Cl(O) \subseteq N_{ano} -sc *g -Cl(U - G) = U - G \subseteq L$. Hence $N_{ano} -sc *g -Cl(O) \subseteq L$.

(iv) $\Rightarrow$ (v). If $K \cap L \neq \emptyset$, where K is non empty and L is N_{ano} -closed, then $K \cap (U - L) \neq \emptyset$ and $U - L$ is N_{ano} -open. Therefore by (iv), there exists $N_{ano} -sc *g$ -open set S such that $K \cap S \neq \emptyset$ and $S \subseteq N_{ano} -sc *g -Cl(S) \subseteq U - L$. Put $P = U - [N_{ano} -sc *g -Cl(S)]$. Then P is $N_{ano} -sc *g$ -open set of U , $U - L \subseteq S$ and $N_{ano} -sc *g -Cl(U - S) \subseteq P$, $U - S \subseteq P$. Hence $L \subseteq P$.

(v) $\Rightarrow$ (i). Let F be a N_{ano} -closed set such that $x \notin F$. Then $\{x\} \cap F = \emptyset$. By (v), there exist disjoint $N_{ano} -sc *g$ -open sets S and P such that $\{x\} \cap S = \emptyset$ and $F \subseteq P$, and hence U is $N_{ano} -sc *g$ -regular.

Definition 4.13. A N_{ano} -Top-Space $(U, T_R(X))$ is called almost N_{ano} -regular space, if for every N_{ano} -regular closed set F and $x \notin F$, there exist disjoint N_{ano} -open sets G and H such that $x \in G$ and $F \subseteq H$.

Definition 4.14. A N_{ano} -Top-Space $(U, T_R(X))$ is called almost $N_{ano} -sc *g$ -regular space, if for every N_{ano} -regular closed set F and $x \notin F$, there exist disjoint $N_{ano} -sc *g$ -open sets G and H such that $x \in G$ and $F \subseteq H$.

Remark 4.15. Every almost N_{ano} -regular space is almost $N_{ano} -sc *g$ -regular and every $N_{ano} -sc *g$ -regular space is almost $N_{ano} -sc *g$ -regular.

Theorem 4.16. Let $(U, T_R(X))$ be any N_{ano} -Top-Space. Then the following statements are equivalent:

(i) U is almost $N_{ano} -sc *g$ -regular.

(ii) For every point $x \in U$ and N_{ano} -regular open set H with $x \in H$ there exists a $N_{ano} -sc *g$ -open set G such that $x \in G \subseteq N_{ano} -sc *g -Cl(G) \subseteq H$.

(iii) For each N_{ano} -regular closed set F of U , $F = \bigcap \{N_{ano} -sc *g -Cl(G) : F \subseteq G \text{ \& } G \in N_{ano} -sc *g -O(U, T_R(X))\}$.

(iv) For every N_{ano} -subset $A \subseteq U$ and each N_{ano} -regular open set H with $A \cap H \neq \emptyset$, there exists $G \in N_{ano} -sc *g -O(U, T_R(X))$ such that $A \cap G \neq \emptyset$ and $N_{ano} -sc *g -Cl(G) \subseteq H$.

(v) For every nonempty N_{ano} -subset A of U and each N_{ano} -regular closed subset F of U with $A \cap F \neq \emptyset$, there exists $G, H \in N_{ano} -sc *g -O(U, T_R(X))$ such that $A \cap G \neq \emptyset$, $F \subseteq H$ and $G \cap H = \emptyset$.

Proof. (i) $\Rightarrow$ (ii). Let H be a N_{ano} -regular open set in U set containing x . Then $x \notin U - H$ and $U - H$ is a N_{ano} -regular closed set in U . Thus by (i), there exist disjoint $N_{ano} -sc *g$ -open sets G and K such that $x \in G$, $U - H \subseteq K$. So, $x \in G \subseteq N_{ano} -sc *g -Cl(G) \subseteq N_{ano} -sc *g -Cl(U - K) = U - K \subseteq H$.

(ii) $\Rightarrow$ (iii). Let F be a N_{ano} -regular closed in U and $x \in U - F$. Then by (ii), there exists $G \in N_{ano} -sc *g -O(U, T_R(X))$ such that $x \in G \subseteq N_{ano} -sc *g -Cl(G) \subseteq U - F$. So, $F \subseteq U - [N_{ano} -sc *g -Cl(G)] = H$ (say) $\in N_{ano} -sc *g -O(U, T_R(X))$ and $G \cap H = \emptyset$. Then $x \notin N_{ano} -sc *g -Cl(H)$. Therefore it follows that $F = \bigcap \{N_{ano} -sc *g -Cl(H) : F \subseteq H \text{ \& } H \in N_{ano} -sc *g -O(U, T_R(X))\}$.



(iii) $\Rightarrow$ (iv). Let A be a N_{ano} -subset of U and H be a N_{ano} -regular open subset of U such that $A \cap H \neq \phi$. Let $x \in A \cap H$. Then $x \notin U - H$. Hence by (iii), there exist $W \in N_{ano}\text{-}sc^*g\text{-}O(U, T_R(X))$ such that $U - H \subseteq W$ and $x \notin N_{ano}\text{-}sc^*g\text{-}Cl(W)$. Put $M = U - [N_{ano}\text{-}sc^*g\text{-}Cl(W)]$. Then M is $N_{ano}\text{-}sc^*g$ -open in U and $x \in M$. Hence $A \cap M \neq \phi$. Now $M \subseteq U - W$, so $N_{ano}\text{-}sc^*g\text{-}Cl(M) \subseteq U - W \subseteq H$.

(iv) $\Rightarrow$ (v). Let F be a set as in the hypothesis of (v). Then $U - F$ is N_{ano} -regular open set in U with $A \cap (U - F) \neq \phi$ and hence by (iv), there exists $N_{ano}\text{-}sc^*g$ -open set G such that $A \cap G \neq \phi$ and $N_{ano}\text{-}sc^*g\text{-}Cl(G) \subseteq U - F$. If we put $H = U - [N_{ano}\text{-}sc^*g\text{-}Cl(G)]$, then H is $N_{ano}\text{-}sc^*g$ -open, $F \subseteq H$ and $G \cap H = \phi$.

(v) $\Rightarrow$ (i). Let F be a N_{ano} -regular closed set in U set not containing x . Then $F \cap \{x\} = \phi$. Then by (v), there exist $N_{ano}\text{-}sc^*g$ -open sets G and H such that $x \in G, F \subseteq H$ and $G \cap H = \phi$.

5. Nano Semi C* Generalized Normal Spaces

In this section, we introduce and study the concepts of $N_{ano}\text{-}sc^*g$ -normal spaces, almost $N_{ano}\text{-}sc^*g$ -normal spaces and mildly $N_{ano}\text{-}sc^*g$ -open spaces via $N_{ano}\text{-}sc^*g$ -open sets in N_{ano} -Top-Spaces.

Definition 5.1. A N_{ano} -Top-Space $(U, T_R(X))$ is called N_{ano} -noraml space if for any two disjoint N_{ano} -closed sets E and F , there exist two disjoint N_{ano} -open sets G and H such that $E \subseteq G$ and $F \subseteq H$.

Definition 5.2. A N_{ano} -Top-Space $(U, T_R(X))$ is called $N_{ano}\text{-}sc^*g$ -noraml space, if for any two disjoint N_{ano} -closed sets E and F , there exist two disjoint $N_{ano}\text{-}sc^*g$ -open sets G and H such that $E \subseteq G$ and $F \subseteq H$.

Theorem 5.3. Every N_{ano} -noraml space is $N_{ano}\text{-}sc^*g$ -noraml space.

Proof. Proof follows from the fact that every N_{ano} -open set is $N_{ano}\text{-}sc^*g$ -open.

Theorem 5.4. Let $f : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be N_{ano} -continuous, bijective, $N_{ano}\text{-}sc^*g$ -open function from a N_{ano} -normal space $(U, T_R(X))$ onto a N_{ano} -Top-Space $(V, T_{R'}(Y))$. Then $(V, T_{R'}(Y))$ is $N_{ano}\text{-}sc^*g$ -noraml.

Proof. Let E and F be disjoint N_{ano} -closed sets in V . Since f is N_{ano} -continuous bijective, $f^{-1}(E)$ and $f^{-1}(F)$ are disjoint N_{ano} -closed sets in U . Now U is a N_{ano} -normal space, there exist disjoint N_{ano} -open sets G and H such that $f^{-1}(E) \subseteq G$ and $f^{-1}(F) \subseteq H$. That is $E \subseteq f(G)$ and $F \subseteq f(H)$. Since f is $N_{ano}\text{-}sc^*g$ -open function, $f(G)$, $f(H)$ are $N_{ano}\text{-}sc^*g$ -open sets in V and f is injective, $f(G) \cap f(H) = f(\phi) = \phi$. Therefore V is a $N_{ano}\text{-}sc^*g$ -noraml space.

Theorem 5.5. Let $f : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be $N_{ano}\text{-}sc^*g$ -continuous, N_{ano} -closed injection and $(V, T_{R'}(Y))$ is a N_{ano} -normal space. Then $(U, T_R(X))$ is $N_{ano}\text{-}sc^*g$ -noraml.

Proof. Let E and F be disjoint N_{ano} -closed sets in U . Since f is N_{ano} -closed injection, $f(E)$ and $f(F)$ are disjoint N_{ano} -closed sets in V . Now V is a N_{ano} -normal space, there exist disjoint N_{ano} -open sets G and H such that $f(E) \subseteq G$ and $f(F) \subseteq H$. That is $E \subseteq f^{-1}(G)$ and $F \subseteq f^{-1}(H)$. Since f is $N_{ano}\text{-}sc^*g$ -continuous function, $f^{-1}(G)$, $f^{-1}(H)$ are $N_{ano}\text{-}sc^*g$ -open sets in U . Further $f^{-1}(G) \cap f^{-1}(H) = f^{-1}(\phi) = \phi$. Therefore U is a $N_{ano}\text{-}sc^*g$ -noraml space.



Theorem 5.6. Let $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be N_{ano} - τ_{sc}^*g -irresolute, N_{ano} -closed injection and $(V, \tau_{R'}(Y))$ is a N_{ano} - τ_{sc}^*g -noraml. Then $(U, \tau_R(X))$ is N_{ano} - τ_{sc}^*g -noraml.

Proof. Let E and F be disjoint N_{ano} -closed sets in U . Since f is N_{ano} -closed injection, $f(E)$ and $f(F)$ are disjoint N_{ano} -closed sets in V . Now V is a N_{ano} - τ_{sc}^*g -noraml space, there exist disjoint N_{ano} - τ_{sc}^*g -open sets G and H such that $f(E) \subseteq G$ and $f(F) \subseteq H$. That is $E \subseteq f^{-1}(G)$ and $F \subseteq f^{-1}(H)$. Since f is N_{ano} - τ_{sc}^*g -irresolute, $f^{-1}(G)$, $f^{-1}(H)$ are N_{ano} - τ_{sc}^*g -open sets in U . Further $f^{-1}(G) \cap f^{-1}(H) = f^{-1}(\phi) = \phi$. Therefore U is a N_{ano} - τ_{sc}^*g -noraml space.

Theorem 5.7. Let $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be N_{ano} - τ_{sc}^*g -continuous bijective, strongly N_{ano} - τ_{sc}^*g -open function and $(U, \tau_R(X))$ is N_{ano} - τ_{sc}^*g -noraml. Then $(V, \tau_{R'}(Y))$ is also N_{ano} - τ_{sc}^*g -noraml.

Proof. Let E and F be disjoint N_{ano} -closed sets in V . Since f is N_{ano} -continuous bijective, $f^{-1}(E)$ and $f^{-1}(F)$ are disjoint N_{ano} -closed sets in U . Now U is a N_{ano} -normal space, there exist disjoint N_{ano} - τ_{sc}^*g -open sets G and H such that $f^{-1}(E) \subseteq G$ and $f^{-1}(F) \subseteq H$. That is $E \subseteq f(G)$ and $F \subseteq f(H)$. Since f is strongly N_{ano} - τ_{sc}^*g -open function, $f(G)$, $f(H)$ are N_{ano} - τ_{sc}^*g -open sets in V and f is injective, $f(G) \cap f(H) = f(\phi) = \phi$. Therefore V is a N_{ano} - τ_{sc}^*g -noraml space.

Theorem 5.8. Let $(U, \tau_R(X))$ be a N_{ano} -Top-Space. Then the following statements are equivalent.

- (i) U is N_{ano} - τ_{sc}^*g -noraml.
- (ii) For each N_{ano} -closed set A and for each N_{ano} -open set G containing A , there exists a N_{ano} - τ_{sc}^*g -open set S containing A such that $N_{ano}\text{-}\tau_{sc}^*g\text{-Cl}(S) \subseteq G$.
- (iii) For each pair of disjoint N_{ano} -closed sets A and B there exists a N_{ano} - τ_{sc}^*g -open set G containing A such that $[N_{ano}\text{-}\tau_{sc}^*g\text{-Cl}(G)] \cap B = \phi$.

Proof. (i) $\Rightarrow$ (ii): Let A be a N_{ano} -closed set and G be a N_{ano} -open set containing A . Then $A \cap (U - G) = \phi$. Thus A and $U - G$ are N_{ano} -closed sets in U . Since U is N_{ano} - τ_{sc}^*g -noraml, there exist disjoint N_{ano} - τ_{sc}^*g -open sets S and H such that $A \subseteq S$, $U - G \subseteq H$ that is $U - H \subseteq G$. Now $S \cap H = \phi$ implies $S \subseteq U - H$. Hence $N_{ano}\text{-}\tau_{sc}^*g\text{-Cl}(S) \subseteq N_{ano}\text{-}\tau_{sc}^*g\text{-Cl}(U - H) = U - H$, because $U - H$ is N_{ano} - τ_{sc}^*g -closed set. Thus, $A \subseteq S \subseteq N_{ano}\text{-}\tau_{sc}^*g\text{-Cl}(S) \subseteq U - H \subseteq G$. That is $N_{ano}\text{-}\tau_{sc}^*g\text{-Cl}(S) \subseteq G$.

(ii) $\Rightarrow$ (iii): Let A and B be disjoint N_{ano} -closed sets in U . Then $A \subseteq U - B$ and $U - B$ is N_{ano} -open set containing A . By (ii), there exists a N_{ano} - τ_{sc}^*g -open set G such that $A \subseteq G$ and $[N_{ano}\text{-}\tau_{sc}^*g\text{-Cl}(G)] \subseteq U - B$, which implies $[N_{ano}\text{-}\tau_{sc}^*g\text{-Cl}(G)] \cap B = \phi$.

(iii) $\Rightarrow$ (i): Let A and B be disjoint N_{ano} -closed sets in U . By (iii), there exists a N_{ano} - τ_{sc}^*g -open set G such that $A \subseteq G$ and $[N_{ano}\text{-}\tau_{sc}^*g\text{-Cl}(G)] \cap B = \phi$ or $B \subseteq U - [N_{ano}\text{-}\tau_{sc}^*g\text{-Cl}(G)]$. Put $H = U - [N_{ano}\text{-}\tau_{sc}^*g\text{-Cl}(G)]$. Now G and H are disjoint N_{ano} - τ_{sc}^*g -open sets of U such that $A \subseteq G$ and $B \subseteq H$. Hence U is N_{ano} - τ_{sc}^*g -noraml.

Theorem 5.9. Let $(U, \tau_R(X))$ be a N_{ano} -Top-Space. Then the following statements are equivalent:

(i) U is N_{ano} - $\neg sc^*g$ -noraml.

(ii) For any two N_{ano} - $\neg sc^*g$ -open sets G and H such that $G \cup H = U$, there exist N_{ano} - $\neg sc^*g$ -closed sets A and B such that $A \subseteq G$ and $B \subseteq H$ and $A \cup B = U$.

Proof. (i) $\Rightarrow$ (ii): Let G and H be two N_{ano} - $\neg sc^*g$ -open sets in U such that $G \cup H = U$. Then G^c and H^c are disjoint N_{ano} - $\neg sc^*g$ -closed sets. Since U is N_{ano} - $\neg sc^*g$ -noraml, then there exist disjoint N_{ano} - $\neg sc^*g$ -open sets P and Q such that $G^c \subseteq P$ and $H^c \subseteq Q$. Put $A = P^c$ and $B = Q^c$. Then A and B are N_{ano} - $\neg sc^*g$ -closed subsets of G and H respectively such that $A \cup B = U$.

(ii) $\Rightarrow$ (i): Let A and B be disjoint N_{ano} - $\neg sc^*g$ -closed sets in U . Then A^c and B^c are N_{ano} - $\neg sc^*g$ -open sets such that $A^c \cup B^c = U$. By (ii), there exists N_{ano} - $\neg sc^*g$ -closed sets E and F such that $E \subseteq A^c$, $F \subseteq B^c$ and $E \cup F = U$. Then $G = E^c$ and $H = F^c$ are disjoint N_{ano} - $\neg sc^*g$ -open sets containing A and B respectively. Therefore U is N_{ano} - $\neg sc^*g$ -noraml.

Definition 5.10. A N_{ano} -Top-Space $(U, \tau_r(X))$ is said to be almost N_{ano} - $\neg sc^*g$ -noraml if for each N_{ano} - $\neg sc^*g$ -closed A and N_{ano} -regular closed B such that $A \cap B = \emptyset$, there exist disjoint N_{ano} - $\neg sc^*g$ -open sets G and H such that $A \subseteq G$ and $B \subseteq H$.

Theorem 5.11. Let $(U, \tau_r(X))$ be a N_{ano} -Top-Space. Then the following statements are equivalent:

(i) U is almost N_{ano} - $\neg sc^*g$ -noraml.

(ii) For each N_{ano} - $\neg sc^*g$ -open set G and for each N_{ano} -regular open set H such that $G \cup H = U$, there exist N_{ano} - $\neg sc^*g$ -closed sets A and B such that $A \subseteq G$ and $B \subseteq H$ and $A \cup B = U$.

(iii) For every N_{ano} - $\neg sc^*g$ -closed set A and N_{ano} -regular open set G containing A , there exists a N_{ano} - $\neg sc^*g$ -open set H such that $A \subseteq H \subseteq [N_{ano}\neg sc^*g\text{-Cl}(H)] \subseteq G$.

Proof. (i) $\Rightarrow$ (ii): Let G be a N_{ano} - $\neg sc^*g$ -open set and H be a N_{ano} -regular open set in U such that $G \cup H = U$. Put $A = U - G$ and $B = U - H$. Then A is a N_{ano} - $\neg sc^*g$ -closed set and B is a N_{ano} -regular closed set in U such that $A \cap B = \emptyset$. Since U is almost N_{ano} - $\neg sc^*g$ -noraml, there exist disjoint N_{ano} - $\neg sc^*g$ -open sets P and Q such that $A \subseteq P$ and $B \subseteq Q$. Put $E = U - P$ and $F = U - Q$. Then E and F are N_{ano} - $\neg sc^*g$ -closed sets such that $E \subseteq G$ and $F \subseteq H$ and $E \cup F = U$.

(ii) $\Rightarrow$ (iii): Let A be N_{ano} - $\neg sc^*g$ -closed set and G be a N_{ano} -regular open set containing A . Then $U - A$ and G are N_{ano} - $\neg sc^*g$ -open set and N_{ano} -regular open set respectively such that $(U - A) \cup G = U$. Then by (ii), there exist N_{ano} - $\neg sc^*g$ -closed sets P and Q such that $P \subseteq (U - A)$ and $Q \subseteq G$ and $P \cup Q = U$. Then $A \subseteq U - P$ and $U - G \subseteq U - Q$ and $(U - P) \cap (U - Q) = \emptyset$. Put $E = U - P$ and $F = U - Q$. Then E and F are disjoint N_{ano} - $\neg sc^*g$ -open sets in U such that $A \subseteq U - P = E \subseteq Q \subseteq G$. Thus we obtain $A \subseteq E \subseteq N_{ano}\neg sc^*g\text{-Cl}(E) \subseteq N_{ano}\neg sc^*g\text{-Cl}(Q) = Q \subseteq G$.

(iii) $\Rightarrow$ (i): Let A be a N_{ano} - $\neg sc^*g$ -closed set and B be a N_{ano} -regular closed set in U such that $A \cap B = \emptyset$. Put $G = U - B$. Then $B \cap G = \emptyset$. We have $A \subseteq G$ where G is a N_{ano} -regular open set. Then by (iii), there exists a N_{ano} - $\neg sc^*g$ -open set H of U such that $A \subseteq H \subseteq [N_{ano}\neg sc^*g\text{-Cl}(H)] \subseteq G$. Put $M = U - [N_{ano}\neg sc^*g\text{-Cl}(H)]$. Then

M is a N_{ano} - sc^*g -open set and $H \cap M = \emptyset$. Hence A and B are separated by N_{ano} - sc^*g -open sets H and M . Therefore U is N_{ano} - sc^*g -normal.

Definition 5.12. A N_{ano} -Top-Space $(U, T_R(X))$ is said to be N_{ano} - sc^*g -Urysohn if for each pair of distinct points $x, y \in U$, there exist N_{ano} - sc^*g -open sets G and H such that $x \in G$, $y \in H$ and $[N_{ano}\text{-}\text{sc}^*g\text{-Cl}(G)] \cap [N_{ano}\text{-}\text{sc}^*g\text{-Cl}(H)] = \emptyset$.

Theorem 5.13. Let $\psi: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be a quasi N_{ano} - sc^*g -irresolute and injective mapping, where V is N_{ano} - sc^*g -Urysohn. Then $(U, T_R(X))$ is N_{ano} - sc^*g - T_2 .

Proof. Let p and q be any two distinct points of U . Since ψ is injective, $\psi(p) \neq \psi(q)$. Again since V is N_{ano} - sc^*g -Urysohn, there exist N_{ano} - sc^*g -open sets K and L in V such that $\psi(p) \in K$, $\psi(q) \in L$ and $[N_{ano}\text{-}\text{sc}^*g\text{-Cl}(K)] \cap [N_{ano}\text{-}\text{sc}^*g\text{-Cl}(L)] = \emptyset$. Again since ψ is quasi N_{ano} - sc^*g -irresolute, there exist N_{ano} - sc^*g -open sets P and Q in U containing p and q respectively such that $\psi(P) \subseteq N_{ano}\text{-}\text{sc}^*g\text{-Cl}(K)$ and $\psi(Q) \subseteq N_{ano}\text{-}\text{sc}^*g\text{-Cl}(L)$. Hence $\psi(P) \cap \psi(Q) \subseteq N_{ano}\text{-}\text{sc}^*g\text{-Cl}(K) \cap N_{ano}\text{-}\text{sc}^*g\text{-Cl}(L) = \emptyset$. Thus $\psi(P) \cap \psi(Q) = \emptyset$ and so $P \cap Q = \emptyset$. Therefore $(U, T_R(X))$ is N_{ano} - sc^*g - T_2 .

Theorem 5.14. Let the function $\psi: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be N_{ano} - sc^*g -open and bijective, where $(U, T_R(X))$ is N_{ano} - sc^*g -Urysohn. Then $(V, T_{R'}(Y))$ is N_{ano} - sc^*g -Urysohn.

Proof. Let p and q be any two distinct points of V . Since ψ is bijective, there exist $a, b \in U$ with $a \neq b$, $\psi(a) = p$ and $\psi(b) = q$. Since $(U, T_R(X))$ is N_{ano} - sc^*g -Urysohn, there exist N_{ano} - sc^*g -open sets G and H in U such that $a \in G$, $b \in H$ and $[N_{ano}\text{-}\text{sc}^*g\text{-Cl}(G)] \cap [N_{ano}\text{-}\text{sc}^*g\text{-Cl}(H)] = \emptyset$. But the N_{ano} - sc^*g -open-ness of ψ ensures that $\psi(U - (N_{ano}\text{-}\text{sc}^*g\text{-Cl}(G))) = V - \psi(N_{ano}\text{-}\text{sc}^*g\text{-Cl}(G))$ and $\psi(U - (N_{ano}\text{-}\text{sc}^*g\text{-Cl}(H))) = V - \psi(N_{ano}\text{-}\text{sc}^*g\text{-Cl}(H))$ are N_{ano} - sc^*g -open sets and so $\psi(N_{ano}\text{-}\text{sc}^*g\text{-Cl}(G))$ and $\psi(N_{ano}\text{-}\text{sc}^*g\text{-Cl}(H))$ are N_{ano} - sc^*g -closed in $(V, T_{R'}(Y))$. Now $[N_{ano}\text{-}\text{sc}^*g\text{-Cl}(\psi(G))] \cap [N_{ano}\text{-}\text{sc}^*g\text{-Cl}(\psi(H))] \subseteq N_{ano}\text{-}\text{sc}^*g\text{-Cl}[\psi(N_{ano}\text{-}\text{sc}^*g\text{-Cl}(G))] \cap N_{ano}\text{-}\text{sc}^*g\text{-Cl}[\psi(N_{ano}\text{-}\text{sc}^*g\text{-Cl}(H))] = \psi(N_{ano}\text{-}\text{sc}^*g\text{-Cl}(G)) \cap \psi(N_{ano}\text{-}\text{sc}^*g\text{-Cl}(H)) = \emptyset$.

Also, since ψ is N_{ano} - sc^*g -open, $\psi(G)$ and $\psi(H)$ are N_{ano} - sc^*g -open sets in V containing p and q respectively. Thus $(V, T_{R'}(Y))$ is N_{ano} - sc^*g -Urysohn.

Theorem 5.15. A function $f: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is strongly N_{ano} - sc^*g -closed if and only if for each subset B of V and for each N_{ano} - sc^*g -open M in U containing $f^{-1}(B)$, there exists a N_{ano} - sc^*g -open set H containing B such that $f^{-1}(H) \subseteq M$.

Proof. Suppose that f is strongly N_{ano} - sc^*g -closed. Let B be a subset of V and $M \in N_{ano}\text{-}\text{sc}^*g\text{-O}(U, T_R(X))$ containing $f^{-1}(B)$. Put $H = V - f(U - M)$, then H is a N_{ano} - sc^*g -open set of V such that $B \subseteq H$ and $f^{-1}(H) \subseteq M$.

Conversely, let K be any N_{ano} - sc^*g -closed set of U . Then $f^{-1}[V - f(K)] \subseteq U - K$ and $U - K \in N_{ano}$ - sc^*g - $O(U, T_R(X))$. There exists a N_{ano} - sc^*g -open set H of V such that $V - f(K) \subseteq H$ and $f^{-1}(H) \subseteq U - K$. Therefore, we have $f(K) \subseteq V - H$ and $K \subseteq f^{-1}(V - H)$. Hence, we obtain $f(K) = V - H$ and $f(K)$ is N_{ano} - sc^*g -closed in V . This shows that f is strongly N_{ano} - sc^*g -closed.

Theorem 5.16. If $f : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is a strongly N_{ano} - sc^*g -closed continuous function from a N_{ano} - sc^*g -normal space U onto a space V , then V is N_{ano} - sc^*g -normal.

Proof. Let H and K be disjoint N_{ano} -closed sets in V . Then $f^{-1}(H)$ and $f^{-1}(K)$ are N_{ano} -closed sets. Since U is N_{ano} - sc^*g -normal, then there exist disjoint N_{ano} - sc^*g -open sets M and L such that $f^{-1}(H) \subseteq M$ and $f^{-1}(K) \subseteq L$. By Theorem 5.15, there exist N_{ano} - sc^*g -open sets A and B such that $H \subseteq A$, $K \subseteq B$, $f^{-1}(A) \subseteq M$ and $f^{-1}(B) \subseteq L$. Also, A and B are disjoint. Thus, V is N_{ano} - sc^*g -normal.

Definition 5.17. A function $f : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is called almost N_{ano} - sc^*g -irresolute if for each x in U and each N_{ano} - sc^*g -neighbourhood M of $f(x)$, N_{ano} - sc^*g - $Cl[f^{-1}(M)]$ is a N_{ano} - sc^*g -neighbourhood of x .

Lemma 5.18. For a function $f : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$, the following statements are equivalent:

(i) f is almost N_{ano} - sc^*g -irresolute.

(ii) $f^{-1}(H) \subseteq N_{ano}$ - sc^*g - $Int[N_{ano}$ - sc^*g - $Cl(f^{-1}(H))]$ for every $H \in N_{ano}$ - sc^*g - $O(V, T_{R'}(Y))$.

Theorem 5.19. A function $f : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is almost N_{ano} - sc^*g -irresolute if and only if $f[N_{ano}$ - sc^*g - $Cl(M)] \subseteq N_{ano}$ - sc^*g - $Cl[f(M)]$ for every $M \in N_{ano}$ - sc^*g - $O(U, T_R(X))$.

Proof. Let $M \in N_{ano}$ - sc^*g - $O(U, T_R(X))$. Suppose $y \notin N_{ano}$ - sc^*g - $Cl[f(M)]$. Then there exists $H \in N_{ano}$ - sc^*g - $O((V, T_{R'}(Y)); y)$ such that $H \cap f(M) = \emptyset$. Hence, $f^{-1}(H) \cap M = \emptyset$. Since $M \in N_{ano}$ - sc^*g - $O(U, T_R(X))$, So, N_{ano} - sc^*g - $Int[N_{ano}$ - sc^*g - $Cl(f^{-1}(H))]$ $\cap N_{ano}$ - sc^*g - $Cl(M) = \emptyset$.

Then by Lemma 5.18, $f^{-1}(H) \cap [N_{ano}$ - sc^*g - $Cl(M)] = \emptyset$ and hence $H \cap f[N_{ano}$ - sc^*g - $Cl(M)] = \emptyset$. This implies that $y \notin f[N_{ano}$ - sc^*g - $Cl(M)]$.

Conversely if $H \in N_{ano}$ - sc^*g - $O(V, T_{R'}(Y))$, then $P = U - [N_{ano}$ - sc^*g - $Cl[f^{-1}(H)]] \in N_{ano}$ - sc^*g - $O(U, T_R(X))$. By assumption, $f[N_{ano}$ - sc^*g - $Cl(P)] \subseteq N_{ano}$ - sc^*g - $Cl[f(P)]$ and so, $U - [N_{ano}$ - sc^*g - $Int(N_{ano}$ - sc^*g - $Cl(f^{-1}(H)))$] = N_{ano} - sc^*g - $Cl(P) \subseteq f^{-1}[N_{ano}$ - sc^*g - $Cl(f(P))]$ $\subseteq f^{-1}[N_{ano}$ - sc^*g - $Cl(f(U - f^{-1}(H)))$] $\subseteq f^{-1}[N_{ano}$ - sc^*g - $Cl(V - H)] \subseteq f^{-1}(V - H) = U - f^{-1}(H)$.

Therefore, it follows that $f^{-1}(H) \subseteq N_{ano}$ - sc^*g - $Int[N_{ano}$ - sc^*g - $Cl(f^{-1}(H))]$. By Lemma 5.18, f is almost N_{ano} - sc^*g -irresolute.

Theorem 5.20. If $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is a strongly N_{ano} - sc^*g -open continuous almost N_{ano} - sc^*g -irresolute function from a N_{ano} - sc^*g -normal space U onto a space V , then V is N_{ano} - sc^*g -normal.

Proof. Let A be a N_{ano} -closed subset of V and B be a N_{ano} -open set containing A . Then by continuity of f , $f^{-1}(A)$ is N_{ano} -closed and $f^{-1}(B)$ is a N_{ano} -open set of U such that $f^{-1}(A) \subseteq f^{-1}(B)$. As U is N_{ano} - sc^*g -normal, there exists a N_{ano} - sc^*g -open set M in U such that $f^{-1}(A) \subseteq M \subseteq N_{ano}\text{-}sc^*g\text{-}Cl(M) \subseteq f^{-1}(B)$ by Theorem 5.7. Then, $f[f^{-1}(A)] \subseteq f(M) \subseteq f[N_{ano}\text{-}sc^*g\text{-}Cl(M)] \subseteq f[f^{-1}(B)]$. Since f is strongly N_{ano} - sc^*g -open almost N_{ano} - sc^*g -irresolute surjection, we obtain $A \subseteq f(M) \subseteq N_{ano}\text{-}sc^*g\text{-}Cl(f(M)) \subseteq B$. Then by Theorem 5.7, the space V is N_{ano} - sc^*g -normal.

Definition 5.21. A function $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is called

- (i) N_{ano} - R -map if $f^{-1}(H)$ is N_{ano} -regular-open in U for every N_{ano} -regular-open set H of V .
- (ii) completely N_{ano} -continuous if $f^{-1}(H)$ is N_{ano} -regular-open in U for every N_{ano} -open set H of V .
- (iii) quasi N_{ano} - sc^*g -closed if $f(A)$ is N_{ano} -closed in V for each $A \in N_{ano}\text{-}sc^*g\text{-}C(U, T_R(X))$.

Theorem 5.22. If $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is a N_{ano} -continuous strongly N_{ano} - sc^*g -open N_{ano} - R -map and almost N_{ano} - sc^*g -irresolute surjection from an almost N_{ano} - sc^*g -normal space U onto a space V , then V is almost N_{ano} - sc^*g -normal.

Proof. Similar to Theorem 5.20.

Corollary 5.23. If $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is a completely N_{ano} -continuous strongly N_{ano} - sc^*g -open and almost N_{ano} - sc^*g -irresolute surjection from an almost N_{ano} - sc^*g -normal space U onto a space V , then V is almost N_{ano} - sc^*g -normal.

Definition 5.24. A N_{ano} -Top-Space $(U, T_R(X))$ is said to be mildly N_{ano} - sc^*g -normal if for every pair of disjoint regular closed sets A and B of U , there exist disjoint N_{ano} - sc^*g -open sets M and H such that $A \subseteq M$ and $B \subseteq H$.

Remark 5.25. The following diagram holds for a N_{ano} -Top-Space $(U, T_R(X))$:

$$\begin{array}{c} N_{ano}\text{-}sc^*g\text{-normal} \Rightarrow \text{almost } N_{ano}\text{-}sc^*g\text{-normal} \Rightarrow \\ \text{mildly } N_{ano}\text{-}sc^*g\text{-normal} \end{array}$$

Theorem 5.26. Let $(U, T_R(X))$ be a N_{ano} -Top-Space. Then the following statements are equivalent:

- (i) U is mildly N_{ano} - sc^*g -normal.
- (ii) For every pair of N_{ano} -regular-open sets G and H , whose union is U , there exist N_{ano} - sc^*g -closed sets A and B such that $A \subseteq G$ and $B \subseteq H$ and $A \cup B = U$.



(iii) For any N_{ano} -regular-closed set A and every N_{ano} -regular-open set B containing A , there exists a N_{ano} - sc^*g -open set H such that $A \subseteq H \subseteq [N_{ano}\text{-}\text{sc}^*g\text{-}Cl(H)] \subseteq B$.

(iv) For every pair of disjoint N_{ano} -regular-closed sets A and B , there exist N_{ano} - sc^*g -open sets G and H such that $A \subseteq G$ and $B \subseteq H$ and $[N_{ano}\text{-}\text{sc}^*g\text{-}Cl(G)] \cap [N_{ano}\text{-}\text{sc}^*g\text{-}Cl(H)] = \emptyset$.

Proof. Similar to Theorem 5.11.

Theorem 5.27. If $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is a strongly N_{ano} - sc^*g -open N_{ano} - R -map and almost N_{ano} - sc^*g -irresolute function from a mildly N_{ano} - sc^*g -normal space U onto a space V , then V is mildly N_{ano} - sc^*g -normal.

Proof. Let A be N_{ano} -regular-closed set and B be a N_{ano} -regular-open set containing A . Then by N_{ano} - R -map of f , $f^{-1}(A)$ is a N_{ano} -regular-closed set contained in the N_{ano} -regular-open set $f^{-1}(B)$. Since U is mildly N_{ano} - sc^*g -normal, there exists a N_{ano} - sc^*g -open set H such that $f^{-1}(A) \subseteq H \subseteq N_{ano}\text{-}\text{sc}^*g\text{-}Cl(H) \subseteq f^{-1}(B)$ by Theorem 5.25. As f is strongly N_{ano} - sc^*g -open and an almost N_{ano} - sc^*g -irresolute surjection, it follows that $f(H) \in N_{ano}\text{-}\text{sc}^*g\text{-}O(V, T_{R'}(Y))$ and $A \subseteq f(H) \subseteq N_{ano}\text{-}\text{sc}^*g\text{-}Cl[f(H)] \subseteq B$. Hence V is mildly N_{ano} - sc^*g -normal.

Theorem 5.28. If $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is N_{ano} - R -map, strongly N_{ano} - sc^*g -closed function from a mildly N_{ano} - sc^*g -normal space U onto a space V , then V is mildly N_{ano} - sc^*g -normal.

Proof. Similar to Theorem 5.20.

Theorem 5.29. Let $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be N_{ano} -continuous quasi N_{ano} - sc^*g -closed surjection and U is N_{ano} - sc^*g -normal, then V is N_{ano} -normal.

Proof. Let G and H be any disjoint N_{ano} -closed sets of V . Since f is N_{ano} -continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint N_{ano} -closed sets of U . Since U is N_{ano} - sc^*g -normal, there exist disjoint $K, L \in N_{ano}\text{-}\text{sc}^*g\text{-}O(U, T_R(X))$ such that $f^{-1}(G) \subseteq K$ and $f^{-1}(H) \subseteq L$. Put $P = V - f(U - K)$ and $Q = V - f(U - L)$, then P and Q are N_{ano} -open sets in V , $G \subseteq P$, $H \subseteq Q$, $f^{-1}(P) \subseteq K$ and $f^{-1}(Q) \subseteq L$. Since $K \cap L = \emptyset$ and f is surjective; we have $P \cap Q = \emptyset$. This shows that V is N_{ano} -normal.

Theorem 5.30. Let $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be a N_{ano} -closed N_{ano} - sc^*g -irresolute injection and V is N_{ano} - sc^*g -normal, then U is N_{ano} - sc^*g -normal.

Proof. Let P and Q be any disjoint N_{ano} -closed sets of U . Since f is N_{ano} -closed injection, $f(P)$ and $f(Q)$ are disjoint N_{ano} -closed sets of V . Since V is N_{ano} - sc^*g -normal, there exist disjoint $G, H \in N_{ano}\text{-}\text{sc}^*g\text{-}O(V, T_{R'}(Y))$ such that $f(P) \subseteq G$ and $f(Q) \subseteq H$. Since f is N_{ano} - sc^*g -irresolute, $f^{-1}(G)$ and $f^{-1}(H)$ are N_{ano} - sc^*g -open sets of U . We also have $P \subseteq f^{-1}(G)$ and $Q \subseteq f^{-1}(H)$ as well as $f^{-1}(G) \cap f^{-1}(H) = \emptyset$. This shows that U is N_{ano} - sc^*g -normal.



Lemma 5.31. A function $f : (U, T_R(X)) \rightarrow (V, T_R(Y))$ is almost N_{ano} - sc^*g -closed if and only if for each subset B of V and each $M \in N_{ano} - R - O(U, T_R(X))$ containing $f^{-1}(B)$, there exists a N_{ano} - sc^*g -open set H of V such that $B \subseteq H$ and $f^{-1}(H) \subseteq M$.

Theorem 5.32. Let $f : (U, T_R(X)) \rightarrow (V, T_R(Y))$ be a N_{ano} -continuous almost N_{ano} - sc^*g -closed surjection. If U is N_{ano} -normal, then V is N_{ano} - sc^*g -normal.

Proof. Let P and Q be any disjoint, N_{ano} -closed sets of V . Since f is N_{ano} -continuous, $f^{-1}(P)$ and $f^{-1}(Q)$ are disjoint N_{ano} -closed sets of U . Since U is N_{ano} -normal, there exist disjoint N_{ano} -open sets K and L such that $f^{-1}(P) \subseteq K$ and $f^{-1}(Q) \subseteq L$. Now put $G = N_{ano} - Int[N_{ano} - Cl(K)]$ and $H = N_{ano} - Int[N_{ano} - Cl(L)]$. Then $G, H \in N_{ano} - R - O(U, T_R(X))$, $f^{-1}(P) \subseteq K \subseteq G$, $f^{-1}(Q) \subseteq L \subseteq H$ and $G \cap H = \emptyset$. By Lemma 5.30, there exists N_{ano} - sc^*g -open sets A and B of V such that $P \subseteq A$, $Q \subseteq B$, $f^{-1}(A) \subseteq K \subseteq G$ and $f^{-1}(B) \subseteq L \subseteq H$. Since $G \cap H = \emptyset$ and f is surjective. We have $A \cap B = \emptyset$. This shows that V is N_{ano} - sc^*g -normal.

6. Nano Semi C* Generalized Compactness

Definition 6.1. A collection $\{K_i : i \in J\}$ of N_{ano} - sc^*g -open sets in N_{ano} -Top-Space $(U, T_R(X))$ is called N_{ano} - sc^*g -open cover of a subset L of $(U, T_R(X))$, if $L \subseteq \bigcup \{K_i : i \in J\}$.

Definition 6.2. A N_{ano} -Top-Space $(U, T_R(X))$ is called N_{ano} - sc^*g -compact if every N_{ano} - sc^*g -open cover of U has a finite subcover.

Definition 6.3. A subset L of N_{ano} -Top-Space $(U, T_R(X))$ is called N_{ano} - sc^*g -compact relative to U if for every collection $\{K_i : i \in J\}$ of N_{ano} - sc^*g -open subsets of U such that $L \subseteq \bigcup \{K_i : i \in J\}$, there exists a finite subset J_0 of J such that $L \subseteq \bigcup \{K_i : i \in J_0\}$.

Definition 6.4. A subset L of N_{ano} -Top-Space $(U, T_R(X))$ is called N_{ano} - sc^*g -compact if L is N_{ano} - sc^*g -compact as a subspace of U .

Theorem 6.5. Every N_{ano} - sc^*g -compact space is N_{ano} -compact space.

Proof. Assume that $(U, T_R(X))$ is a N_{ano} - sc^*g -compact space. Let $\Gamma = \{K_i : i \in I\}$ be a N_{ano} -open cover of U . Since every N_{ano} -open set is N_{ano} - sc^*g -open. Hence Γ is a N_{ano} - sc^*g -open cover of U . By hypothesis, Γ has a finite sub covering $\Gamma^* = \{K_i : i=1,2,...,n\}$ of U . Hence $(U, T_R(X))$ is N_{ano} -compact.

Theorem 6.6. Every N_{ano} - sc^*g -closed subset of N_{ano} - sc^*g -compact space is N_{ano} - sc^*g -compact relative to U .

Proof. Let U be N_{ano} - sc^*g -compact space and K be N_{ano} - sc^*g -closed subset of U . Then $U - K$ is N_{ano} - sc^*g -open set in U . Let $S = \{K_i : i \in J\}$ be N_{ano} - sc^*g -open cover of K by N_{ano} - sc^*g -open subsets of U . Then $S^* = S \cup \{U - K\}$ is a N_{ano} - sc^*g -open cover of U . Since U is N_{ano} - sc^*g -compact, S^* is reducible to a



finite subcover of U say $U = K_{i_1} \cup K_{i_2} \cup \dots \cup K_{i_n} \cup (U - K)$, $K_{i_k} \in S$. But K and $U - K$ are disjoint. Hence $K \subseteq K_{i_1} \cup K_{i_2} \cup \dots \cup K_{i_n}$, $K_{i_k} \in S$. This implies that any N_{ano} - sc^*g -open cover S of K contains a finite subcover. Therefore K is N_{ano} - sc^*g -compact relative to U .

Theorem 6.7. If $f: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is surjective, N_{ano} - sc^*g -continuous function and U is N_{ano} - sc^*g -compact then V is N_{ano} -compact.

Proof. Let $f: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be surjective, N_{ano} - sc^*g -continuous from N_{ano} - sc^*g -compact space $(U, T_R(X))$ to N_{ano} -Top-Space $(V, T_{R'}(Y))$. Let $\{K_i: i \in J\}$ be N_{ano} -open cover of V . Since f is N_{ano} - sc^*g -continuous, $\{f^{-1}(K_i): i \in J\}$ is N_{ano} - sc^*g -open cover of U . Since U is N_{ano} - sc^*g -compact implies N_{ano} - sc^*g -open cover $\{f^{-1}(K_i): i \in J\}$ has a finite subcover say $\{f^{-1}(K_1), f^{-1}(K_2), \dots, f^{-1}(K_n)\}$. Since f is surjective $\{K_1, K_2, \dots, K_n\}$ is a N_{ano} -open cover of V , which is finite. Hence V is N_{ano} -compact.

Theorem 6.8. If $f: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is N_{ano} - sc^*g -irresolute and a subset L of U is N_{ano} - sc^*g -compact subset relative to U , then the image $f(L)$ is N_{ano} - sc^*g -compact relative to V .

Proof. Let $\{K_i: i \in J\}$ be any collection of N_{ano} - sc^*g -open subsets of V such that $f(L) \subseteq \bigcup \{K_i: i \in J\}$. Then $L \subseteq \bigcup \{f^{-1}(K_i): i \in J\}$ where $\{f^{-1}(K_i): i \in J\}$ is family of N_{ano} - sc^*g -open sets in U . Since L is N_{ano} - sc^*g -compact relative to U , the N_{ano} - sc^*g -open cover $\{f^{-1}(K_i): i \in J\}$ of U has a finite subcover say $\{f^{-1}(K_i): i \in J_0\}$, where J_0 is a finite subset of J such that $L \subseteq \bigcup \{f^{-1}(K_i): i \in J_0\}$. Thus $f(L) \subseteq \bigcup \{K_i: i \in J_0\}$. Hence $f(L)$ is N_{ano} - sc^*g -compact relative to V .

Theorem 6.9. A N_{ano} -Top-Space $(U, T_R(X))$ is N_{ano} - sc^*g -compact if and only if every family of N_{ano} - sc^*g -closed sets of $(U, T_R(X))$ having finite intersection property has a non-empty intersection.

Proof. Suppose $(U, T_R(X))$ is N_{ano} - sc^*g -compact. Let $\gamma = \{G_i: i \in I\}$ be a family of N_{ano} - sc^*g -closed sets of $(U, T_R(X))$ with finite intersection property. Suppose $\bigcap_{i \in I} G_i = \emptyset$. Then $U - \bigcap_{i \in I} G_i = U$. This implies $\bigcup_{i \in I} (U - G_i) = U$. Thus the cover $\Gamma = \{U - G_i: i \in I\}$ is a N_{ano} - sc^*g -open cover of $(U, T_R(X))$. Since $(U, T_R(X))$ is N_{ano} - sc^*g -compact. Therefore the N_{ano} - sc^*g -open cover Γ has a finite subcover say $\Lambda = \{U - G_i: i = 1, 2, \dots, n\}$. This implies $U = \bigcup_{i=1}^n (U - G_i)$ which implies $U = U - \bigcap_{i=1}^n G_i$ which implies $\bigcap_{i=1}^n G_i = \emptyset$. This contradicts the assumption. Hence $\bigcap_{i \in I} G_i \neq \emptyset$.

Conversely, suppose $(U, T_R(X))$ is not N_{ano} - sc^*g -compact. Then there exists a N_{ano} - sc^*g -open cover of $(U, T_R(X))$ say $\Omega = \{H_i: i \in I\}$ having no finite subcover. This implies for any finite subfamily $\Omega_n = \{H_i: i = 1, 2, \dots, n\}$ of Ω , we have $\bigcup_{i=1}^n H_i \neq U$, which implies $\bigcup_{i=1}^n (U - H_i) \neq \emptyset$. Then the family $\square = \{U - H_i: i \in I\}$ of N_{ano} - sc^*g -closed has a finite intersection property. Also, by assumption $\bigcap_{i \in I} (U - H_i) \neq \emptyset$ which implies $U - \bigcup_{i \in I} H_i \neq \emptyset$ so that $\bigcup_{i \in I} H_i \neq U$. This implies $\Omega = \{H_i: i \in I\}$ is not a cover for $(U, T_R(X))$. This contradicts the fact that $\Omega = \{H_i: i \in I\}$ is a cover for $(U, T_R(X))$. Therefore any N_{ano} - sc^*g -open cover $\Omega = \{H_i: i \in I\}$ of $(U, T_R(X))$ has a finite subcover $\Omega_n = \{H_i: i = 1, 2, \dots, n\}$. Hence $(U, T_R(X))$ is N_{ano} - sc^*g -compact.

Definition 6.10. A N_{ano} -Top-Space $(U, T_R(X))$ is said to be N_{ano} -Lindelof space if every N_{ano} -open cover of $(U, T_R(X))$ has a countable subcover.

Definition 6.11. A N_{ano} -Top-Space $(U, T_R(X))$ is said to be N_{ano} -sc* g -Lindelof space if every N_{ano} -sc* g -open cover of $(U, T_R(X))$ has a countable subcover.

Theorem 6.12. Every N_{ano} -sc* g -compact space $(U, T_R(X))$ is N_{ano} -sc* g -Lindelof space.

Proof. Let $(U, T_R(X))$ be N_{ano} -sc* g -compact space. Let $\{G_i : i \in I\}$ be a N_{ano} -sc* g -open cover of $(U, T_R(X))$. Then $\{G_i : i \in I\}$ has a finite subcover $\{G_i : i = 1, 2, \dots, n\}$, since $(U, T_R(X))$ is N_{ano} -sc* g -compact. Since every finite subcover is always a countable subcover and therefore, $\{G_i : i = 1, 2, \dots, n\}$ is countable subcover of $\{G_i : i \in I\}$ for $(U, T_R(X))$. Hence $(U, T_R(X))$ is N_{ano} -sc* g -Lindelof space.

Theorem 6.13. If $f : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is surjective N_{ano} -sc* g -continuous function and U is N_{ano} -sc* g -Lindelof space, then V is N_{ano} -Lindelof space.

Proof. Let $f : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be a N_{ano} -sc* g -continuous map from a N_{ano} -sc* g -Lindelof space $(U, T_R(X))$ onto a N_{ano} -Top-Space $(V, T_{R'}(Y))$. Let $\{H_i : i \in I\}$ be a N_{ano} -open cover of $(V, T_{R'}(Y))$, then $\{f^{-1}(H_i) : i \in I\}$ is a N_{ano} -sc* g -open cover of $(U, T_R(X))$, since f is N_{ano} -sc* g -continuous. As $(U, T_R(X))$ is N_{ano} -sc* g -Lindelof, the N_{ano} -sc* g -open cover $\{f^{-1}(H_i) : i \in I\}$ of $(U, T_R(X))$ has a countable subcover $\{f^{-1}(H_i) : i \in I_0\}$ for some countable subset I_0 of I . Therefore $U = \bigcup_{i \in I_0} f^{-1}(H_i)$ which implies $f(U) = V = \bigcup_{i \in I_0} H_i$. Consequently $\{H_i : i \in I_0\}$ is a countable subcover of $\{H_i : i \in I\}$ for $(V, T_{R'}(Y))$. Hence $(V, T_{R'}(Y))$ is N_{ano} -Lindelof space.

Definition 6.14. A N_{ano} -Top-Space $(U, T_R(X))$ is called countably N_{ano} -sc* g -compact if every countable N_{ano} -sc* g -open cover of U has a finite subcover.

Theorem 6.15. If a N_{ano} -Top-Space $(U, T_R(X))$ is N_{ano} -sc* g -Lindelof and countably N_{ano} -sc* g -compact space, then $(U, T_R(X))$ is N_{ano} -sc* g -compact.

Proof. Suppose $(U, T_R(X))$ is N_{ano} -sc* g -Lindelof and countably N_{ano} -sc* g -compact space. Let $\{G_i : i \in I\}$ be a N_{ano} -sc* g -open cover of $(U, T_R(X))$. Since $(U, T_R(X))$ is N_{ano} -sc* g -Lindelof. Therefore $\{G_i : i \in I\}$ has a countable subcover $\{H_n = G_{i_n} : n \in N\}$. Then $\{H_n = G_{i_n} : n \in N\}$ is a countable N_{ano} -sc* g -open cover of $(U, T_R(X))$. Again since $(U, T_R(X))$ is countably N_{ano} -sc* g -compact space. Hence $\{H_n = G_{i_n} : n \in N\}$ has a finite subcover $\{H_{n_k} : k = 1, 2, \dots, n\}$. consequently $\{H_{n_k} : k = 1, 2, \dots, n\}$ is a finite subcover of $\{G_i : i \in I\}$ for $(U, T_R(X))$. Hence $(U, T_R(X))$ is N_{ano} -sc* g -compact space.

Theorem 6.16. If $f : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is N_{ano} -sc* g -irresolute function and a subset W of U is N_{ano} -sc* g -compact relative to U , then the image $f(W)$ is N_{ano} -sc* g -compact relative to V .



Proof. Let $\{H_i : i \in I\}$ be any collection of N_{ano} - sc^*g -open sets in V such that $f(W) \subseteq \bigcup_{i \in I} H_i$. Then $W \subseteq \bigcup_{i \in I} f^{-1}(H_i)$, where $\{f^{-1}(H_i) : i \in I\}$ is a family of N_{ano} - sc^*g -open sets in U . Since W is N_{ano} - sc^*g -compact relative to U , there exists a finite subcollection $\{H_i : i = 1, 2, \dots, n\}$ such that $W \subseteq \bigcup_{i=1}^n f^{-1}(H_i)$. Hence $f(W)$ is N_{ano} - sc^*g -compact relative to V .

Corollary 6.17. If $f : (U, T_R(X)) \rightarrow (V, T_R(Y))$ is N_{ano} - sc^*g -irresolute function and a subset W of U is N_{ano} - sc^*g -Lindelof relative to U , then the image $f(W)$ is N_{ano} - sc^*g -Lindelof relative to V .

7. Nano Semi C^* Generalized Connectedness

In this section, we study N_{ano} - sc^*g -connected spaces in N_{ano} -Top-Spaces. We also study the relationship between N_{ano} -connectedness and N_{ano} - sc^*g -connectedness and some properties.

Definition 7.1. A N_{ano} -Top-Space $(U, T_R(X))$ is said to be N_{ano} -connected if $(U, T_R(X))$ cannot be expressed as a disjoint union of two non-empty N_{ano} -open sets. A subset of $(U, T_R(X))$ is N_{ano} -connected as a subspace. A subset is said to be N_{ano} -disconnected if and only if it is not N_{ano} -connected.

Definition 7.2. Two non-empty subsets A and B in a N_{ano} -Top-Space $(U, T_R(X))$ are said to be N_{ano} - sc^*g -separated if $A \cap N_{ano}\text{-}sc^*g\text{-}Cl(B) = B \cap N_{ano}\text{-}sc^*g\text{-}Cl(A) = \phi$.

Definition 7.3. A N_{ano} -Top-Space $(U, T_R(X))$ is called N_{ano} - sc^*g -connected if there is not a N_{ano} - sc^*g -separation of U . Otherwise, it is called N_{ano} - sc^*g -disconnected.

Definition 7.4. A subset A of a N_{ano} -Top-Space $(U, T_R(X))$ is called N_{ano} - sc^*g -connected if there is no a N_{ano} - sc^*g -separation of A . Otherwise, it is called N_{ano} - sc^*g -disconnected.

Theorem 7.5. If G and H are N_{ano} - sc^*g -separated and $A \subseteq G$, $B \subseteq H$, then A and B are also N_{ano} - sc^*g -separated.

Proof. Let G and H be N_{ano} - sc^*g -separated. Then $G \cap N_{ano}\text{-}sc^*g\text{-}Cl(H) = H \cap N_{ano}\text{-}sc^*g\text{-}Cl(G) = \phi$. Since $A \subseteq G$ and $B \subseteq H$, $N_{ano}\text{-}sc^*g\text{-}Cl(A) \subseteq N_{ano}\text{-}sc^*g\text{-}Cl(G)$ and $N_{ano}\text{-}sc^*g\text{-}Cl(B) \subseteq N_{ano}\text{-}sc^*g\text{-}Cl(H)$ which implies, $A \cap [N_{ano}\text{-}sc^*g\text{-}Cl(B)] \subseteq G \cap [N_{ano}\text{-}sc^*g\text{-}Cl(H)] = \phi$ and hence $A \cap [N_{ano}\text{-}sc^*g\text{-}Cl(B)] = \phi$. Similarly $B \cap [N_{ano}\text{-}sc^*g\text{-}Cl(A)] = \phi$. Therefore A and B are N_{ano} - sc^*g -separated.

Theorem 7.6. Each two N_{ano} - sc^*g -separated sets are always disjoint.

Proof. Let G and H be N_{ano} - sc^*g -separated. Then $G \cap [N_{ano}\text{-}sc^*g\text{-}Cl(H)] = H \cap [N_{ano}\text{-}sc^*g\text{-}Cl(G)] = \phi$. Now $G \cap H \subseteq G \cap [N_{ano}\text{-}sc^*g\text{-}Cl(H)] = \phi$. Then $G \cap H = \phi$. Hence G and H are disjoint.

Theorem 7.7. If G and H are N_{ano} - sc^*g -open and if $A = G \cap (U - H)$ and $B = H \cap (U - G)$, then A and B are N_{ano} - sc^*g -separated.



Proof. Let G and H be N_{ano} - sc^*g -open subsets of U . Then $U - G$ and $U - H$ are N_{ano} - sc^*g -closed. Since $A \subseteq U - H$, $N_{ano}-sc^*g-CI(A) \subseteq N_{ano}-sc^*g-CI(U - H) = U - H$ and so $[N_{ano}-sc^*g-CI(A)] \cap H = \emptyset$. Since $B \subseteq H$, $[N_{ano}-sc^*g-CI(A)] \cap B = \emptyset$. Similarly, $[N_{ano}-sc^*g-CI(B)] \cap A = \emptyset$. Hence A and B are N_{ano} - sc^*g -separated.

Theorem 7.8. Any two disjoint non-empty N_{ano} - sc^*g -closed subsets of a N_{ano} -Top-Space $(U, T_R(X))$ are N_{ano} - sc^*g -separated.

Proof. Suppose G and H are disjoint non-empty N_{ano} - sc^*g -closed sets. Then $G \cap N_{ano}-sc^*g-CI(H) = H \cap N_{ano}-sc^*g-CI(G) = \emptyset$. Hence G and H are N_{ano} - sc^*g -separated.

Theorem 7.9. A N_{ano} -Top-Space $(U, T_R(X))$ is N_{ano} - sc^*g -connected if U cannot be expressed as a disjoint union of two non-empty N_{ano} - sc^*g -open sets in U .

Proof. Let U be N_{ano} - sc^*g -connected. Assume that two disjoint non-empty N_{ano} - sc^*g -open subsets of U such that $U = G \cup H$. Take $A = U - G$ and $B = U - H$. Then A and B are N_{ano} - sc^*g -closed in U . Therefore $A \cap N_{ano}-sc^*g-CI(B) = B \cap N_{ano}-sc^*g-CI(A) = \emptyset$ and $U = A \cup B$. Thus, U is not N_{ano} - sc^*g -connected, but this is a contradiction with the hypothesis. Thus, U cannot be expressed as the union of two disjoint non-empty N_{ano} - sc^*g -open subsets of U .

Conversely, suppose that the condition holds. Let $U = A \cup B$, $A \neq \emptyset$, $B \neq \emptyset$ and $A \cap [N_{ano}-sc^*g-CI(B)] = B \cap [N_{ano}-sc^*g-CI(A)] = \emptyset$. Take $G = U - [N_{ano}-sc^*g-CI(A)]$ and $H = U - [N_{ano}-sc^*g-CI(B)]$. Then G and H are non-empty N_{ano} - sc^*g -open sets and $G \cup H = [U - (N_{ano}-sc^*g-CI(A))] \cup [U - (N_{ano}-sc^*g-CI(B))] = U - ([N_{ano}-sc^*g-CI(A)] \cap [N_{ano}-sc^*g-CI(B)])$. $\cap (U - ([N_{ano}-sc^*g-CI(A)] \cap [N_{ano}-sc^*g-CI(B)])) \subseteq U$. This implies that $U = G \cup H$. Again $G \cap H = [U - (N_{ano}-sc^*g-CI(A))] \cap [U - (N_{ano}-sc^*g-CI(B))] = (N_{ano}-sc^*g-CI(A)) \cup (N_{ano}-sc^*g-CI(B)) = \emptyset$, but this is a contradiction with the hypothesis. Therefore, U is N_{ano} - sc^*g -connected.

Definition 7.10. A N_{ano} -Top-Space $(U, T_R(X))$ is called N_{ano} - sc^*g -connected if U is not the disjoint union of two nonempty N_{ano} - sc^*g -open subsets.

Theorem 7.11. If $f : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is N_{ano} - sc^*g -continuous, surjection and U is N_{ano} - sc^*g -connected then V is N_{ano} -connected.

Proof. Suppose V is not N_{ano} -connected. Then $V = K \cup L$ where K and L are disjoint nonempty N_{ano} -open sets in V . Since f is N_{ano} - sc^*g -continuous surjection, $U = f^{-1}(K) \cup f^{-1}(L)$ where $f^{-1}(K)$ and $f^{-1}(L)$ are disjoint nonempty N_{ano} - sc^*g -open subsets of U implies U is not N_{ano} - sc^*g -connected space. This is a contradiction to the hypothesis. Therefore V is N_{ano} -connected.

Theorem 7.12. If $f : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is N_{ano} - sc^*g -irresolute, surjection and U is N_{ano} - sc^*g -connected then V is also N_{ano} - sc^*g -connected.

Proof. Suppose V is not N_{ano} - sc^*g -connected. Then $V = K \cup L$ where K and L are disjoint nonempty N_{ano} - sc^*g -open sets in V . Since f is N_{ano} - sc^*g -irresolute surjection, $U = f^{-1}(K) \cup f^{-1}(L)$ where $f^{-1}(K)$



and $f^{-1}(\mathbb{L})$ are disjoint nonempty N_{ano} -sc $\star g$ -open subsets of U implies U is not N_{ano} -sc $\star g$ -connected space. This is a contradiction to the hypothesis. Therefore V is N_{ano} -sc $\star g$ -connected.

Corollary 7.13. If $f:(U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ is bijective N_{ano} -sc $\star g$ -open function and V is N_{ano} -sc $\star g$ -connected then U is N_{ano} -connected.

Theorem 7.14. Let $(U, T_R(X))$ be a N_{ano} -Top-Space. Then the following statements are equivalent.

- (i) U is N_{ano} -sc $\star g$ -connected.
- (ii) The only subsets of U which are both N_{ano} -sc $\star g$ -open and N_{ano} -sc $\star g$ -closed are the empty set ϕ and U .
- (iii) Each N_{ano} -sc $\star g$ -continuous function of U into a discrete space V with at least two points is a constant function.

Proof. (i) $\Rightarrow$ (ii). Suppose (i) holds and F is a proper subset of U which is both N_{ano} -sc $\star g$ -open and N_{ano} -sc $\star g$ -closed. Then $U - F$ is also both N_{ano} -sc $\star g$ -open and N_{ano} -sc $\star g$ -closed. Therefore $U = F \cup (U - F)$ is disjoint union of two nonempty N_{ano} -sc $\star g$ -open sets. This contradicts the fact that U is N_{ano} -sc $\star g$ -connected. Hence $F = \phi$ or U .

(ii) $\Rightarrow$ (i). Suppose (ii) holds. If possible U is not N_{ano} -sc $\star g$ -connected, then $U = K \cup L$ where K and L are disjoint nonempty N_{ano} -sc $\star g$ -open sets in U . Since $K = U - L$, K is N_{ano} -sc $\star g$ -closed set. But by assumption, $K = \phi$ or U which is contradiction. Hence (i) hold.

(ii) $\Rightarrow$ (iii). Let f be N_{ano} -sc $\star g$ -continuous function, where V is discrete space with at least two points. Then $f^{-1}(\{m\})$ is both N_{ano} -sc $\star g$ -open and N_{ano} -sc $\star g$ -closed for each $m \in V$ and $U = \bigcup \{f^{-1}(\{m\}) : m \in V\}$. By assumption, $f^{-1}(\{m\}) = U$ or ϕ . If $f^{-1}(\{m\}) = \phi$ for all $m \in V$, then f will not be a function. Also there cannot exist more than one point $m \in V$ such that $f^{-1}(\{m\}) = U$ and $f^{-1}(\{m_1\}) = \phi$ where $m \neq m_1 \in V$. This shows that f is a constant function.

(iii) $\Rightarrow$ (ii). Let F be both N_{ano} -sc $\star g$ -open and N_{ano} -sc $\star g$ -closed in U . Suppose $F \neq \phi$ and f be a N_{ano} -sc $\star g$ -continuous function defined by $f(F) = \{l\}$ and $f(U - F) = \{m\}$ for some distinct points l and m in V . By assumption f is constant function. Therefore $F = U$.

Theorem 7.15. If a N_{ano} -Top-Space $(U, T_R(X))$ is N_{ano} -sc $\star g$ -connected, then U is also N_{ano} -connected space.

Proof. Let U be N_{ano} -sc $\star g$ -connected. Then the only N_{ano} -sc $\star g$ -clopen subsets of U are ϕ and U . Suppose U is not N_{ano} -connected. Then there exists a non-empty proper subset W of U which is N_{ano} -clopen in U . Since every N_{ano} -clopen is also N_{ano} -sc $\star g$ -clopen, W is also N_{ano} -sc $\star g$ -clopen in U . Hence W is a non-empty proper N_{ano} -sc $\star g$ -clopen in U , which is a contradiction. Therefore, U is N_{ano} -connected.

Theorem 7.16. Let $(W, T_R(X))$ and $(W, T_{R'}(Y))$ be two N_{ano} -Top-Spaces on W with $T_{R'}(Y) \subseteq T_R(X)$. If the N_{ano} -Top-Space $(U, T_R(X))$ is N_{ano} -sc $\star g$ -connected, then $(W, T_{R'}(Y))$ is also N_{ano} -sc $\star g$ -connected.



Proof. Suppose that $(W, T_R(X))$ is N_{ano} - sc^*g -connected but $(W, T_{R'}(Y))$ is not. Then $T_{R'}(Y)$ contains a non-empty proper N_{ano} - sc^*g -clopen. Since $T_{R'}(Y) \subseteq T_R(X)$, so also $T_R(X)$ contains N_{ano} - sc^*g -clopen, hence $(W, T_R(X))$ is not N_{ano} - sc^*g -connected, which is contradiction. Therefore, $(W, T_{R'}(Y))$ is also N_{ano} - sc^*g -connected.

Theorem 7.17. Let A be a N_{ano} - sc^*g -connected set of a N_{ano} -Top-Space $(U, T_R(X))$ and G, H are N_{ano} - sc^*g -separated subsets of U such that $A \subseteq G \cup H$. Then either $A \subseteq G$ or $A \subseteq H$.

Proof. Since $A = (A \cap G) \cup (A \cap H)$, we have $(A \cap G) \cap [N_{ano}-sc^*g-Cl(A \cap H)] \subseteq G \cap [N_{ano}-sc^*g-Cl(H)] = \phi$. Similarly, we have $(A \cap H) \cap [N_{ano}-sc^*g-Cl(A \cap G)] \subseteq H \cap [N_{ano}-sc^*g-Cl(G)] = \phi$. If $A \cap G$ and $A \cap H$ are non-empty, then A is not N_{ano} - sc^*g -connected, which is a contradiction. Therefore, either $A \cap G = \phi$ or $A \cap H = \phi$. Therefore, either $A \subseteq G$ or $A \subseteq H$.

Theorem 7.18. If A is a N_{ano} - sc^*g -connected set of a N_{ano} -Top-Space $(U, T_R(X))$ and $A \subseteq B \subseteq N_{ano}-sc^*g-Cl(A)$, then B is N_{ano} - sc^*g -connected.

Proof. Assume that B is not N_{ano} - sc^*g -connected. Then there exist N_{ano} - sc^*g -separated sets G and H such that $B = G \cup H$. Then G and H are non-empty and $G \cap [N_{ano}-sc^*g-Cl(H)] = \phi = H \cap [N_{ano}-sc^*g-Cl(G)]$, then by Theorem 7.17, we have either $A \subset G$ or $A \subseteq H$. So, we have two cases: (i). Suppose $A \subseteq G$. Then $N_{ano}-sc^*g-Cl(A) \subseteq N_{ano}-sc^*g-Cl(G)$ and $H \cap [N_{ano}-sc^*g-Cl(A)] = \phi$. By hypothesis, $G \cup H \subseteq N_{ano}-sc^*g-Cl(A)$, that is $H = \phi$, but this is a contradiction to the fact that H is non-empty.

(ii). Suppose $A \subseteq H$. Using similar argument of part (i), G is empty and this is a contradiction. Thus, B is N_{ano} - sc^*g -connected.

Corollary 7.19. Let A be a N_{ano} - sc^*g -connected subset of a N_{ano} -Top-Space $(U, T_R(X))$. Then $N_{ano}-sc^*g-Cl(A)$ is N_{ano} - sc^*g -connected.

Theorem 7.20. Let A and B be subsets of a N_{ano} -Top-Space $(U, T_R(X))$. If A and B are N_{ano} - sc^*g -connected sets and are not N_{ano} - sc^*g -separated in U , then $A \cup B$ is N_{ano} - sc^*g -connected.

Proof. Suppose $A \cup B$ is not N_{ano} - sc^*g -connected. Then there exist N_{ano} - sc^*g -separated sets G, H in U such that $A \cup B = G \cup H$, then $A \subseteq G \cup H$. By Theorem 7.17, either $A \subseteq G$ or $A \subseteq H$. Again either $B \subseteq G$ or $B \subseteq H$. If $A \subseteq G$ and $B \subseteq G$, then $A \cup B \subseteq G$ and $H = \phi$, which is a contradiction. Therefore $A \subseteq G$ and $B \subseteq H$. Similarly, $A \subseteq H$ and $B \subseteq G$. Thus, $B \cap [N_{ano}-sc^*g-Cl(A)] \subseteq H \cap [N_{ano}-sc^*g-Cl(G)] = \phi$ and $A \cap [N_{ano}-sc^*g-Cl(B)] \subseteq H \cap [N_{ano}-sc^*g-Cl(G)] = \phi$. Hence A, B are N_{ano} - sc^*g -separated in U , which is a contradiction. Therefore, $A \cup B$ is N_{ano} - sc^*g -connected.

8. Nano Semi C* Generalized Closed and Strongly Nano Semi C* Generalized Closed Graphs

Definition 8.1. Let $\psi: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be a function. Then its graph $G(\psi)$ is called N_{ano} - sc^*g -closed if for each $(x, y) \in U \times V - G(\psi)$, there exist a $K \in N_{ano}-sc^*g-O(U, T_R(X); x)$ and a $L \in N_{ano}-sc^*g-O(V, T_{R'}(Y))$ such that $(K \times L) \cap G(\psi) = \phi$.

Lemma 8.2. Let $\psi: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be a function. Then the following conditions are equivalent:

- (i). ψ has N_{ano} -SC* $\mathcal{G}$ -closed graph.
- (ii). For each $(x, y) \in U \times V - G(\psi)$, there exist a $K \in N_{ano}$ -SC* $\mathcal{G}$ - $O(U, T_R(X); x)$ and a $L \in N_{ano}$ -SC* $\mathcal{G}$ - $O(V, T_{R'}(Y); y)$ such that $\psi(U) \times L = \emptyset$.

Proof. The proof is straightforward and thus omitted.

Theorem 8.3. Let $\psi: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be a N_{ano} -SC* $\mathcal{G}$ -irresolute, where V is N_{ano} -SC* $\mathcal{G}$ - T_2 . Then the graph $G(\psi)$ is N_{ano} -SC* $\mathcal{G}$ -closed.

Proof. Let $(x, y) \in U \times V - G(\psi)$. Then $y \neq \psi(x)$. Since V is N_{ano} -SC* $\mathcal{G}$ - T_2 , there exist a $K \in N_{ano}$ -SC* $\mathcal{G}$ - $O(V, T_{R'}(Y); \psi(x))$ and a $L \in N_{ano}$ -SC* $\mathcal{G}$ - $O(V, T_{R'}(Y); y)$ such that $K \cap L = \emptyset$. Also, since ψ is N_{ano} -SC* $\mathcal{G}$ -irresolute, $H = \psi^{-1}(K) \in N_{ano}$ -SC* $\mathcal{G}$ - $O(U, T_R(X); x)$ and so $\psi(H) \cap L = \emptyset$. Therefore the graph $G(\psi)$ is N_{ano} -SC* $\mathcal{G}$ -closed.

Theorem 8.4. Let $\psi: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be a surjective mapping having N_{ano} -SC* $\mathcal{G}$ -closed graph. Then $(V, T_{R'}(Y))$ is N_{ano} -SC* $\mathcal{G}$ - T_1 .

Proof. Let $p, q \in V$ with $p \neq q$. Since ψ is surjective, there exist $a \in U$ such that $\psi(a) = p$ and $\psi(a) \neq q$. Then $(a, q) \in U \times V - G(\psi)$ and so by the Lemma 8.2, we can find a $K \in N_{ano}$ -SC* $\mathcal{G}$ - $O(U, T_R(X); a)$ and a $L \in N_{ano}$ -SC* $\mathcal{G}$ - $O(V, T_{R'}(Y); q)$ such that $\psi(K) \cap L = \emptyset$. Thus $p \in \psi(K)$ and $p \notin L$. Similarly, we can ensure the existence of $b \in U$ such that $\psi(b) = q$ and $\psi(b) \neq p$ and a $P \in N_{ano}$ -SC* $\mathcal{G}$ - $O(U, T_R(X); b)$ and a $Q \in N_{ano}$ -SC* $\mathcal{G}$ - $O(V, T_{R'}(Y); p)$ such that $\psi(P) \cap Q = \emptyset$. Then $q \in \psi(P)$ and $q \notin Q$. So $(V, T_{R'}(Y))$ is N_{ano} -SC* $\mathcal{G}$ - T_1 .

Theorem 8.5. Let $\psi: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be N_{ano} -SC* $\mathcal{G}$ -open and surjective mapping. If the graph $G(\psi)$ is N_{ano} -SC* $\mathcal{G}$ -closed, then $(V, T_{R'}(Y))$ is N_{ano} -SC* $\mathcal{G}$ - T_2 .

Proof. Let $p, q \in V$ with $p \neq q$. Since ψ is surjective, there exist $a \in U$ such that $\psi(a) = p$ and $\psi(a) \neq q$. Then $(a, q) \in U \times V - G(\psi)$ and so by Lemma 8.2, we can find a $K \in N_{ano}$ -SC* $\mathcal{G}$ - $O(U, T_R(X); a)$ and a $L \in N_{ano}$ -SC* $\mathcal{G}$ - $O(V, T_{R'}(Y); q)$ such that $\psi(K) \cap L = \emptyset$. Again since ψ is N_{ano} -SC* $\mathcal{G}$ -open, $\psi(K) \in N_{ano}$ -SC* $\mathcal{G}$ - $O(V, T_{R'}(Y); p)$. So $(V, T_{R'}(Y))$ is N_{ano} -SC* $\mathcal{G}$ - T_2 .

Theorem 8.6. Let $\psi: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be an injective function. If the graph $G(\psi)$ is N_{ano} -SC* $\mathcal{G}$ -closed, then $(U, T_R(X))$ is N_{ano} -SC* $\mathcal{G}$ - T_1 .

Proof. Let $p, q \in U$ with $p \neq q$. Since ψ is injective, $\psi(p) \neq \psi(q)$. So $(p, \psi(q)) \in U \times V - G(\psi)$. Now by Lemma 8.2, there exist a $K \in N_{ano}$ -SC* $\mathcal{G}$ - $O(U, T_R(X); p)$ and a $L \in N_{ano}$ -SC* $\mathcal{G}$ - $O(V, T_{R'}(Y); \psi(q))$ such that $\psi(K) \cap L = \emptyset$. Therefore $\psi(q) \notin \psi(K)$ and so $q \notin K$. Hence $p \in K$ but $q \notin K$. Again since



$(q, \psi(p)) \in U \times V - G(\psi)$, we can find $M \in N_{ano} - SC * G - O(U, T_R(X); q)$ and $p \notin L$. Hence $(U, T_R(X))$ is $N_{ano} - SC * G - T_1$.

Theorem 8.7. Let $\psi: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be a $N_{ano} - SC * G$ -irresolute injection. If the graph $G(\psi)$ is $N_{ano} - SC * G$ -closed, then $(U, T_R(X))$ is $N_{ano} - SC * G - T_2$.

Proof. Let $p, q \in U$ with $p \neq q$. Since ψ is injective, $\psi(p) \neq \psi(q)$. So $(p, \psi(q)) \in U \times V - G(\psi)$. Then Lemma 8.2 ensures the existence of a $K \in N_{ano} - SC * G - O(U, T_R(X); p)$ and a $L \in N_{ano} - SC * G - O(V, T_{R'}(Y); \psi(q))$ such that $\psi(K) \cap L = \emptyset$. Since ψ is $N_{ano} - SC * G$ -irresolute, $\psi^{-1}(L) \in N_{ano} - SC * G - O(U, T_R(X); q)$. So $(U, T_R(X))$ is $N_{ano} - SC * G - T_2$.

Definition 8.8. Let $\psi: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be a function. Then its graph $G(\psi)$ is called strongly $N_{ano} - SC * G$ -closed if for each $(x, y) \in U \times V - G(\psi)$, there exists a $K \in N_{ano} - SC * G - O(U, T_R(X); x)$ and a $L \in N_{ano} - SC * G - O(V, T_{R'}(Y); y)$ such that $[U \times N_{ano} - SC * G - Cl(L)] \cap G(\psi) = \emptyset$. Clearly, every function possessing strongly $N_{ano} - SC * G$ -closed graph has $N_{ano} - SC * G$ -closed graph.

Lemma 8.9. Let $\psi: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be a function. Then the following conditions are equivalent:

(i). The graph $G(\psi)$ is strongly $N_{ano} - SC * G$ -closed.

(ii). For each $(x, y) \in U \times V - G(\psi)$, there exist a $K \in N_{ano} - SC * G - O(U, T_R(X); x)$ and a $L \in N_{ano} - SC * G - O(V, T_{R'}(Y); y)$ such that $\psi(K) \cap [N_{ano} - SC * G - Cl(L)] = \emptyset$.

Proof. The proof is straightforward and thus omitted.

Definition 8.10. A filter base Ω on a N_{ano} -Top-Space $(U, T_R(X))$ is said to $N_{ano} - SC * G$ -converge (respectively $N_{ano} - SC * G$ -converge) to a point $x \in U$ if for each $L \in N_{ano} - SC * G - O(U, T_R(X); x)$, there exists a $K \in \Omega$ such that $K \subseteq N_{ano} - SC * G - Cl(L)$ (respectively $K \subseteq L$).

Theorem 8.11. Let $\psi: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be any function and $(V, T_{R'}(Y))$ be $N_{ano} - SC * G$ -regular. Then the following statements are equivalent:

(i). The graph $G(\psi)$ is strongly $N_{ano} - SC * G$ -closed.

(ii). If a filter base Ω on a N_{ano} -Top-Space $(U, T_R(X))$, $N_{ano} - SC * G$ -converges to x and $\psi(\Omega)$ $N_{ano} - SC * G$ -converges to y in $(V, T_{R'}(Y))$, then $y = \psi(x)$.

Proof. (i) $\Rightarrow$ (ii). Let Ω be a filter base on $(U, T_R(X))$ that $N_{ano} - SC * G$ -converges to x and $\psi(\Omega)$ $N_{ano} - SC * G$ -converges to y . If possible, let $y \neq \psi(x)$. Then $(x, y) \in U \times V - G(\psi)$. Clearly, $N_{ano} - SC * G - O(U, T_R(X); x) \subseteq \Omega$ and $\{N_{ano} - SC * G - Cl(L) : L \in N_{ano} - SC * G - O(V, T_{R'}(Y); y)\} \subseteq \psi(\Omega)$. So for each $K \in N_{ano} - SC * G - O(U, T_R(X); x)$ and each $L \in N_{ano} - SC * G - O(V, T_{R'}(Y); y)$, there exist $P \in \Omega$ and $Q \in \Omega$ such that $P \subseteq K$ and $\psi(Q) \subseteq N_{ano} - SC * G - Cl(L)$. Hence there exists a $M \in \Omega$ such that $M \subseteq P \cap Q$ and satisfies

$M \subseteq K$ as well as $\psi(M) \subseteq N_{ano} -sc \star g -Cl(L)$. Hence $\phi \neq \psi(M) \subseteq \psi(K) \cap [N_{ano} -sc \star g -Cl(L)]$. So by Lemma 8.10, $G(\psi)$ is not strongly $N_{ano} -sc \star g$ -closed.

(i) $\Rightarrow$ (ii). Let $(V, T_{R'}(Y))$ be $N_{ano} -sc \star g$ -regular and the given condition (ii) holds for ψ . If possible, let $G(\psi)$ be not strongly $N_{ano} -sc \star g$ -closed. Then there exists $(x, y) \in U \times V - G(\psi)$ such that $(U \times [N_{ano} -sc \star g -Cl(L)]) \cap G(\psi) \neq \phi$ for each $K \in N_{ano} -sc \star g -O(U, T_R(X); x)$ and each $L \in N_{ano} -sc \star g -O(V, T_{R'}(Y); y)$. Since V is $N_{ano} -sc \star g$ -regular, the family $\Omega = \{M_{KL} = \{p \in K : (p, \psi(p)) \in (K \times N_{ano} -sc \star g -Cl(L)) \cap G(\psi)\} : K \in N_{ano} -sc \star g -O(U, T_R(X); x) \& L \in N_{ano} -sc \star g -O(V, T_{R'}(Y); y)\}$

is a filter base on $(U, T_R(X))$. But Ω $N_{ano} -sc \star g$ -converges to x in $(U, T_R(X))$ and $\psi(\Omega)$ $N_{ano} -sc \star g$ -converges to y and $y = \psi(x)$ – a contradiction.

Theorem 8.12. Let $\psi : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be any function. If the graph $G(\psi)$ is strongly $N_{ano} -sc \star g$ -closed, then for each $x \in U$, $\psi(x) = \bigcap \{N_{ano} -sc \star g -Cl(\psi(K)) : K \in N_{ano} -sc \star g -O(U, T_R(X); x)\}$

Proof. If possible, let there exist an $x \in U$ and an $y (\neq \psi(x)) \in V$ such that $y \in N_{ano} -sc \star g -Cl(\psi(M))$ for each $M \in N_{ano} -sc \star g -O(U, T_R(X); x)$. Since $(x, y) \in U \times V - G(\psi)$, by Lemma 8.9, we can find a $K \in N_{ano} -sc \star g -O(U, T_R(X); x)$ and a $L \in N_{ano} -sc \star g -O(V, T_{R'}(Y); y)$ such that $\psi(K) \cap [N_{ano} -sc \star g -Cl(L)] = \phi$ and so $\psi(K) \cap L \subseteq \psi(K) \cap [N_{ano} -sc \star g -Cl(L)] = \phi$. Thus $\psi(K) \cap L = \phi$. Then $y \notin N_{ano} -sc \star g -Cl(\psi(K))$, a contradiction.

Theorem 8.13. Let $(U, T_R(X))$ be an arbitrary N_{ano} -Top-Space and $(V, T_{R'}(Y))$ is $N_{ano} -sc \star g -T_2$. Let $\psi : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be $N_{ano} -sc \star g$ -irresolute. Then the graph $G(\psi)$ is strongly $N_{ano} -sc \star g$ -closed.

Proof. Let $(x, y) \in U \times V - G(\psi)$. Then $y \neq \psi(x)$. Since $(V, T_{R'}(Y))$ is $N_{ano} -sc \star g -T_2$, there exist a $K \in N_{ano} -sc \star g -O(U, T_R(X); \psi(x))$ and a $L \in N_{ano} -sc \star g -O(V, T_{R'}(Y); y)$ such that $K \cap L = \phi$. Again since ψ is $N_{ano} -sc \star g$ -irresolute, $M = \psi^{-1}(K) \in N_{ano} -sc \star g -O(U, T_R(X); x)$ and so $\psi(M) \cap L = \phi$, i.e. $L \subseteq V - \psi(M)$, i.e. $N_{ano} -sc \star g -Cl(L) \subseteq V - \psi(M)$, i.e. $\psi(M) \cap [N_{ano} -sc \star g -Cl(L)] = \phi$. Hence $G(\psi)$ is strongly $N_{ano} -sc \star g$ -closed.

Theorem 8.14. Let $\psi : (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be a quasi $N_{ano} -sc \star g$ -irresolute injection. If the graph $G(\psi)$ is strongly $N_{ano} -sc \star g$ -closed, then $(U, T_R(X))$ is $N_{ano} -sc \star g -T_2$.

Proof. Let any two distinct points $p, q \in U$. Since ψ is injective, $\psi(p) \neq \psi(q)$. Then $(p, \psi(q)) \in U \times V - G(\psi)$ and so by Lemma 8.9, there exist a $K \in N_{ano} -sc \star g -O(U, T_R(X); p)$ and a $L \in N_{ano} -sc \star g -O(V, T_{R'}(Y); \psi(q))$ such that $\psi(K) \cap [N_{ano} -sc \star g -Cl(L)] = \phi$ and so $\psi^{-1}[N_{ano} -sc \star g -Cl(L)] \subseteq U - K$. Since ψ is a quasi $N_{ano} -sc \star g$ -irresolute, there exist $M \in N_{ano} -sc \star g -O(U, T_R(X); q)$ such that $\psi(M) \subseteq N_{ano} -sc \star g -Cl(L)$. Then $M \subseteq \psi^{-1}[N_{ano} -sc \star g -Cl(L)] \subseteq U - K$ and hence $M \cap K = \phi$. So $(U, T_R(X))$ is $N_{ano} -sc \star g -T_2$.

Theorem 8.15. Let $(U, T_R(X))$ be any N_{ano} -Top-Space, $(V, T_{R'}(Y))$ be N_{ano} -sc* g -Urysohn N_{ano} -Top-Space and $\psi: (U, T_R(X)) \rightarrow (V, T_{R'}(Y))$ be a quasi N_{ano} -sc* g -irresolute. Then its graph $G(\psi)$ is strongly N_{ano} -sc* g -closed.

Proof. Let $(x, y) \in U \times V - G(\psi)$. Then $y \neq \psi(x)$. Since $(V, T_{R'}(Y))$ is N_{ano} -sc* g -Urysohn, there exist a $K \in N_{ano}$ -sc* g - $O(V, T_{R'}(Y); \psi(x))$ and a $L \in N_{ano}$ -sc* g - $O(V, T_{R'}(Y); y)$ such that $[N_{ano}\text{-sc*}g\text{-Cl}(K)] \cap [N_{ano}\text{-sc*}g\text{-Cl}(L)] = \emptyset$. Again since ψ is quasi N_{ano} -sc* g -irresolute, there exists a $H \in N_{ano}$ -sc* g - $O(U, T_R(X); x)$ such that $\psi(H) \subseteq N_{ano}\text{-sc*}g\text{-Cl}(K) \subseteq V - [N_{ano}\text{-sc*}g\text{-Cl}(L)]$. Therefore $\psi(H) \cap [N_{ano}\text{-sc*}g\text{-Cl}(L)] = \emptyset$. Hence $G(\psi)$ is strongly N_{ano} -sc* g -closed.

CONCLUSIONS

We have introduced the notions of N_{ano} -sc* g -Separation Axioms namely N_{ano} -sc* g - T_0 space, N_{ano} -sc* g - T_1 space, N_{ano} -sc* g - T_2 space, N_{ano} -sc* g -regular space, N_{ano} -sc* g -normal space, almost N_{ano} -sc* g -normal space, Mildly N_{ano} -sc* g -normal space, N_{ano} -sc* g -compact space, N_{ano} -sc* g -connected space, N_{ano} -sc* g -closed graph and strongly N_{ano} -sc* g -closed graph in Nano topological spaces. We have investigated fundamental properties and characterizations of these spaces in Nano topological spaces. I intend to continue to investigate further properties and characterizations of N_{ano} -sc* g -continuous, N_{ano} -sc* g -irresolute, N_{ano} -sc* g -open, and N_{ano} -sc* g -closed mappings in Nano Topological Spaces. This work will be expanded in the future with some real-life applications.

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