



Steady Casson Fluid Flow and Heat Transfer Over a Stretching Sheet with Magnetic Induction

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ABSTRACT

This study examines steady boundary-layer flow and heat transfer of an electrically conducting Casson fluid over a linearly stretching sheet with magnetic-induction feedback. A divergence-free magnetic-flux representation yields coupled momentum and induction equations, and a quadratic wall-temperature distribution yields a similarity energy equation. Inclination is represented through the normal component of the applied field. The seven-order boundary-value system is solved by fourth-order collocation. Analytical benchmarks, domain-extension checks and an independent thermal shooting calculation assess numerical accuracy. The reported domain extension from $\eta_{\max} = 160$ to 320 changes the reduced skin-friction coefficient by less than 2.1×10^{-10} and the reduced Nusselt number by less than 1.2×10^{-11} . Increasing the magnetic parameter M from 0.5 to 3 increases the magnitude of reduced wall shear from 1.825687 to 2.234109 and decreases reduced heat transfer from 1.441909 to 1.407584. Magnetic Prandtl number affects velocity and temperature through electromagnetic feedback, while heat absorption and increasing Prandtl number enhance reduced heat transfer. The results apply to a steady, fully yielded effective-viscosity approximation with negligible dissipation and prescribed wall-normal magnetic flux.

Keywords: Casson fluid; magnetic induction; stretching sheet; magnetic Prandtl number; heat absorption; boundary-layer flow.

1. INTRODUCTION

The linearly stretching sheet is a canonical model of transport near continuously moving surfaces (Crane, 1970). Casson-fluid studies extend this setting to an effective non-Newtonian viscosity and prescribed thermal conditions. Mukhopadhyay et al. (2013) investigated an unsteady stretching surface, while Mahanta and Shaw (2015) considered three-dimensional porous-sheet flow. These models motivate careful specification of the geometry and approximation before results are compared.

Magnetic-field orientation changes the normal component that acts on tangential motion. Kalaivanan et al. (2015) examined aligned-field Casson flow with slip, and Devi et al. (2020) considered aligned magnetic forcing with inclined outer velocity. An imposed-field approximation can neglect induction when the appropriate magnetic Reynolds number is small; Abd El-Aziz and Afify (2019) use such an approximation in a Casson-flow setting. Including induction at finite magnetic diffusivity instead requires a solenoidal magnetic field and its associated momentum feedback.

El-Mistikawy (2018) developed stretching-sheet flow with magnetic induction and heat transfer, including magnetic-pressure effects. The present study adopts that consistency requirement in a fully yielded Casson-viscosity approximation and reports reproducible parameter sweeps for a specified wall-normal-flux boundary condition. The contribution is the coupled formulation, explicit wall-quantity definitions and verified

numerical data for this configuration. It is not a claim that induced-field stretching-sheet flow is unexplored. Numerical reproducibility is especially relevant to the methods surveyed by Verma and Mondal (2021).

Table 1. Symbols and dimensionless parameters

Symbol	Definition	Units or retained values
x, y	Coordinates along and normal to the sheet	m
$u, v; Uw$	Velocity components; sheet speed $Uw = cx$	ms^{-1}
c	Stretching rate	s^{-1}
$\rho; \nu$	Density; kinematic viscosity	$kgm^{-3}; m^2s^{-1}$
$\sigma; \mu m$	Electrical conductivity; magnetic permeability	$Sm^{-1}; Hm^{-1}$
$k; cp$	Thermal conductivity; specific heat	$Wm^{-1}K^{-1}; Jkg^{-1}K^{-1}$
$\eta m; \kappa$	Magnetic diffusivity $1/(\mu m \sigma)$; thermal diffusivity $k/(\rho cp)$	m^2s^{-1}
$B_0; B_n; B_x, B_y$	Applied magnitude; $B_0 \sin \phi$; local field components	T
$\psi; \Psi B$	Velocity stream function; magnetic-flux potential	$m^2s^{-1}; Tm$
$T; T_w; T_\infty$	Fluid, wall and ambient temperatures	K
$\Delta T_{ref}; L_{ref}$	Reference wall-temperature excess; reference length	$K; m$
Q_0	Volumetric source coefficient, $Q_0 \leq 0$	$Wm^{-3}K^{-1}$
$\beta; K\beta$	Casson parameter; effective-viscosity factor $1 + 1/\beta$	$\beta = 0.3, 0.5, 0.8, 2$
ϕ	Applied-field angle measured from the sheet	$30^\circ, 45^\circ, 60^\circ, 90^\circ$
$M; N$	$\sigma B_0^2/(\rho c)$; effective magnetic parameter $M \sin^2 \phi$	$M = 0.5, 1, 1.5, 3$
$Pm; Pr$	$\nu/\eta m; \nu/\kappa$	$Pm = 0.35 - 1; Pr = 0.71 - 7$
$H; Gw$	$Q_0/(\rho cp c)$; prescribed normal-field ratio at the wall	$H = -0.5, -0.2, 0; Gw = 1$
$\eta; a$	Similarity coordinates ay ; inverse length $\sqrt{(c/\nu)}$	Dimensionless; m^{-1}
$f; g; \theta$	Similarity stream function, normal-field ratio and temperature	Dimensionless; prime = $d/d\eta$
$Rex; Rmx; Ec$	$cx^2/\nu; PmRex; c^2L_{ref}^2/(cp\Delta T_{ref})$	Dimensionless
$\tau w; qw$	Wall shear stress; wall heat flux	$Pa; Wm^{-2}$
$Cf^*; Nu^*$	$K\beta f''(0); -\theta'(0)$	Reduced dimensionless wall measures
$k^*; A^*$	Decay rate and amplitude in the exact magnetic benchmark	Dimensionless

2. METHODOLOGY

2.1 Physical assumptions and dimensional equations

An impermeable sheet occupies $y = 0$ and stretches tangentially with speed $Uw = cx$, where $x > 0$ and $c > 0$. A laminar, incompressible, electrically conducting fluid occupies $y > 0$. Density ρ , electrical conductivity, mag-



netic permeability μ_m , kinematic viscosity ν , thermal conductivity k and specific heat c_p are constant. The fully yielded Casson approximation replaces the dynamic viscosity by $\rho\nu K\beta$, where $\beta > 0$ is the dimensionless Casson parameter. Yield surfaces and an unyielded plug region are not resolved.

$$U_w = cx, \quad K_\beta = 1 + \frac{1}{\beta^\alpha}, \quad \eta_m = \frac{1}{\mu_m \sigma}, \quad K = \frac{k}{\rho c_p} \quad (1)$$

The applied magnetic-field magnitude is B_0 , and ϕ is measured from the sheet. The model retains the normal component $B_n = B_0 \sin \phi$. A uniform tangential imposed component is excluded from the similarity representation. Inclination therefore modifies the retained normal component; the formulation does not resolve the complete oblique-field geometry. The induced field may have both tangential and normal components.

The dimensional boundary-layer equations are given below. Subscripts denote partial derivatives. The leading normal momentum balance incorporates pressure induced by the tangential magnetic field, with constant external pressure p_∞ . Magnetic tension terms containing streamwise derivatives of B_y vanish in the similarity representation. Streamwise viscous, magnetic and thermal diffusion are neglected under the assumption of a sufficiently large local Reynolds number.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0. \quad (2)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{B_y B_{xy}}{\rho \mu_m} + \frac{B_x^2}{2\mu_m} = p_\infty. \quad (3)$$

Writing the magnetic field in terms of the flux potential Ψ enforces magnetic solenoidality. The steady induction and temperature equations are

$$u \Psi_{B,x} + v \Psi_{B,y} = \eta_m \Psi_{B,yy}, \quad B_x = \Psi_{B,y}, \quad B_y = -\Psi_{B,x}. \quad (4)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \kappa \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p} (T - T_\infty). \quad (5)$$

The electric-field gauge in equation (4) is fixed by the steady exterior state; no externally imposed out-of-plane electric field is included. Q_0 is the coefficient of a volumetric source proportional to temperature excess, with units $Wm^{-3}K^{-1}$. The retained cases use $Q_0 \leq 0$, corresponding to heat absorption or zero source. Buoyancy, viscous heating and Joule heating are omitted under the low-dissipation condition in Section 2.3. Changes in temperature therefore cannot be attributed to electrical-resistance heating.

2.2 Similarity transformation and coupled system

Define the velocity stream function by $u = \psi y$ and $v = -\psi x$. The similarity variables and magnetic-flux representation are

$$\eta = ay, \quad a = \sqrt{\frac{\epsilon}{\nu}}, \quad \psi = \sqrt{\nu \tau} x f(\eta), \quad u = cx f', \quad v = -\sqrt{\nu \tau} f. \quad (6)$$

$$\Psi_B = -B_n x g(\eta), \quad B_x = -B_n x a g', \quad B_y = B_n g, \quad B_n = B_0 \sin \phi. \quad (7)$$

Equation (7) gives $\partial B_x / \partial x = -B_n a g'$ and $\partial B_y / \partial y = B_n a g'$, whose sum is identically zero. The tangential field has units of tesla because xa is dimensionless. The normal flux at the sheet is prescribed, while the tangential field is determined by the solution. Primes denote differentiation with respect to η throughout.

The prescribed wall-temperature excess is quadratic in x : $T_w - T_\infty = \Delta T_{ref} (x/L_{ref})^2$, where $\Delta T_{ref} > 0$



and $L_{ref} > 0$. This scaling gives

$$T = T_{\infty} + \Delta T_{ref} \left(\frac{x}{L_{ref}} \right)^2 \theta(\eta). \quad (8)$$

$$M = \frac{\sigma B_0^2}{\rho c}, \quad N = M \sin^2 \phi, \quad Pm = \frac{\nu}{\eta_m}, \quad Pr = \frac{\nu}{\kappa}, \quad H = \frac{Q_0}{\rho c_p c}. \quad (9)$$

Pm is the magnetic Prandtl number, the ratio of momentum diffusivity to magnetic diffusivity. The local magnetic Reynolds number is $Rmx = Pm Rex$. The effective magnetic parameter N combines field strength and inclination. Substitution into equations (3) – (5) gives

$$K_{\beta} f''' + ff'' - (f')^2 + \frac{N}{Pm} [(g')^2 - gg''] = 0. \quad (10)$$

$$g'' = Pm(fg - fg'). \quad (11)$$

$$\frac{\theta''}{Pr} + f\theta' - 2f'\theta + H\theta = 0. \quad (12)$$

The contribution $(N/Pm)(g')^2$ in equation (10) arises from the streamwise magnetic-pressure gradient, while $(N/Pm)gg''$ arises from the streamwise Lorentz force. Their signs and coefficients are retained together. The term $2f'\theta$ in equation (12) follows from the quadratic wall-temperature variation. Temperature does not feed back into velocity or induction in this forced-convection model.

2.3 Boundary conditions and wall quantities

No slip and no penetration are imposed at the sheet. A magnetically permeable boundary supplies the prescribed normal field $B(x, 0) = BnGw$, and B_y approaches Bn outside the layer. All parameter sweeps use $Gw = 1$. The boundary conditions are

$f(0) = 0, \quad f'(0) = 1, \quad g(0) = Gw = 1,$	$\theta(0) = 1.$	(13)
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$f'(\infty) = 0, \quad g(\infty) = 1, \quad \theta(\infty) = 0.$		(14)
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These seven scalar conditions close the third-order momentum, second-order induction and second-order thermal equations. No additional condition $g'(0) = 0$ is imposed because the wall tangential field is generally nonzero; that extra condition would over determine the system. The reported solutions also recover $g' \rightarrow 0$ in the far field. Prescribed normal flux is a physical boundary condition and does not imply an insulating wall.

The signed local skin-friction coefficient uses $\rho U w^2$ as its denominator; the convention based on $\rho U w^2 / 2$ would give twice this value. Dimensional stress and heat flux are distinguished from their reduced dimensionless measures:

$$Cf_x = \frac{\tau_{wx}}{\rho U w^2}, \quad C_{f,x} = \frac{\tau_{wx}}{\rho U_w^2}, \quad \tau_w = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0}, \quad C_f^* = \sqrt{\frac{K_{\beta} - C_{f,x}}{C_{f,x}}} = \frac{K_{\beta}}{\beta} \quad (15)$$

$$q_w = -kT \left. \frac{\partial \theta}{\partial y} \right|_{y=0}, \quad Nu_x = \frac{xq_w}{k(T_w - T_{\infty})}, \quad Nu^* = \frac{Nu_x}{\sqrt{Re_x}} = -\theta'(0). \quad (16)$$

Negative Cf^* denotes resisting wall stress. Its magnitude is used when discussing resistance. Neither $f''(0)$ nor Cf^* is a dimensional stress. For the prescribed quadratic wall-temperature variation, the Eckert number is constant. A sufficient small-dissipation scaling, assuming bounded similarity derivatives, is

$$Ec = \frac{c^2 L_{reF}^2}{c_p \Delta T_{reF}} \quad Ec \max\{K_\beta, N/Pm^2\} \ll 1. \quad (17)$$

The local Reynolds number must be large enough for the velocity, thermal and magnetic layers to remain thin relative to x . The dimensionless sweeps do not establish a dimensional operating range or material calibration. The streamwise origin $x = 0$ is excluded from these local measures.

2.4 Numerical solution and verification

The first-order state is $(f, f', f'', g, g', \theta, \theta')$. SciPy 1.17.0 `solve_bvp` uses fourth-order collocation with residual control (SciPy community, n.d.). Each reported case uses $\eta_{\max} = 160$, an initial uniform mesh of 801 points, adaptive refinement, residual and boundary tolerances of 10^{-8} , and a maximum of 50,000 nodes. Exponential velocity and temperature guesses, together with $g = 1$, initialise the solver. Every retained case terminates successfully.

The baseline is $M = 1, \phi = 45^\circ, \beta = 0.5, Pm = 0.5, Pr = 1, H = 0$ and $Gw = 1$. Each sweep varies one parameter while retaining the others at baseline. The reported reruns use $\eta_{\max} = 320$ with 1,601 initial points. Maximum domain-extension changes are 2.043×10^{-10} in Cf^* , 1.159×10^{-11} in Nu^* and 1.309×10^{-10} in $g'(0)$. These checks assess the reported numerical branch without establishing global uniqueness.

Table 2 presents the baseline domain and tolerance checks. RMS residual denotes the maximum normalised root-mean-square residual returned by `solve_bvp`. A separately sampled ODE defect measures the absolute discrepancy between the derivative of the continuous numerical solution and the governing first-order system.

Table 2. Baseline domain and tolerance checks

$\eta_{\max}$	Initial nodes	Tolerance	Cf^*	Nu^*	Max. RMS residual
40	401	$1e-06$	-1.915565317	1.434847610	$9.8e-07$
80	401	$1e-08$	-1.915564176	1.434847664	$1.0e-08$
120	601	$1e-08$	-1.915564176	1.434847664	$1.0e-08$
160	801	$1e-08$	-1.915564176	1.434847664	$1.0e-08$
160	1601	$1e-09$	-1.915564176	1.434847664	$1.0e-09$

For $\eta_{\max} = 160$ and tolerance 10^{-8} , the reported maximum sampled absolute ODE defect is 1.64×10^{-8} . Tightening the tolerance to 10^{-9} changes the wall quantities by less than 2×10^{-12} . Boundary residuals are reported below the prescribed tolerance.

At zero field, equation (10) reduces to the exponential stretching-sheet solution (Crane, 1970), modified by the viscosity factor $K\beta$. For $H = 0$ and $Pr = 2/K\beta$, the corresponding exact thermal solution is

$$f' = e^{-\eta/\sqrt{K\beta}}, \quad f = \sqrt{K\beta} (1 - e^{-\eta/\sqrt{K\beta}}), \quad \theta = e^{-2\eta/\sqrt{K\beta}} \quad (N = H = 0, Pr = 2/K\beta). \quad (18)$$

The zero-field benchmark covers $\beta = 0.3, 0.5$ and 2 . The full nonlinear magnetic coupling is also checked using the exact exponential family below, which satisfies equations (10) and (11):

$$f' = e^{-k_*\eta}, \quad f = (1 - e^{-k_*\eta})/k_*, \quad g = 1 + A_*e^{-k_*\eta}, \quad A_* = \frac{Pm}{k_*^2 - Pm}. \quad (19)$$

$$K \frac{k_*^4}{\beta} - (K \frac{Pm}{\beta} + 1 + N) \frac{k_*^2}{\beta} + Pm = 0, \quad G_w = 1 + A_* \quad (20)$$

Use the larger positive root for $k_* \geq 2$. For $K\beta = 3$, $Pm = 0.1$ and $N = 0.5$, the benchmark gives $k_* = 0.7335172897$, $A_* = 0.2282856857$ and $G_w = 1.2282856857$. This prescribed wall flux differs from $G_w = 1$ in the parameter sweeps. Table 3 consolidates the reported analytical and independent-method comparisons.

Table 3. Analytical and independent-method benchmark comparisons

Benchmark	Conditions or compared values	Reported agreement
Zero-field velocity	Equation (18); $N = H = 0$; $\beta = 0.3, 0.5, 2$	Max. absolute profile error $< 3 \times 10^{-10}$
Zero-field temperature	Equation (18); $Pr = 2/K\beta$	Max. absolute profile error $< 7 \times 10^{-11}$
Coupled magnetic solution	Equations (19)–(20); $K\beta = 3$, $Pm = 0.1$, $N = 0.5$; $G_w = 1.2282856857$	Velocity error: 4.19×10^{-10} ; magnetic-profile error: 9.56×10^{-11}
Thermal shooting vs collocation	Baseline Nu_* : 1.434847664146(shooting); 1.434847664161(collocation)	Absolute difference: 1.5×10^{-11}

3. ANALYSIS AND RESULTS

Table 4 presents the reported dimensionless wall quantities, rounded to six decimal places. A single baseline row replaces the repeated baseline entries. Each subsequent row varies only the indicated parameter; all other parameters retain the baseline values.

Table 4. Dimensionless wall quantities for one-parameter sweeps

Varied parameter	Value	$f''(0)$	Cf_*	Nu_*	$g'(0)$
Baseline	—	−0.638521	−1.915564	1.434848	−0.502696
M	0.5	−0.608562	−1.825687	1.441909	−0.497420
M	1.5	−0.667155	−2.001464	1.427847	−0.507788
M	3	−0.744703	−2.234109	1.407584	−0.521740
$\phi(^{\circ})$	30	−0.608562	−1.825687	1.441909	−0.497420
$\phi(^{\circ})$	60	−0.667155	−2.001464	1.427847	−0.507788
$\phi(^{\circ})$	90	−0.694410	−2.083230	1.420947	−0.512674
β	0.3	−0.523118	−2.266846	1.464745	−0.539238
β	0.8	−0.746289	−1.679151	1.406470	−0.474019
β	2	−0.929321	−1.393982	1.357088	−0.434004
Pm	0.35	−0.648168	−1.944503	1.431861	−0.390857



Pm	0.7	-0.629531	-1.888592	1.437501	-0.634381
Pm	1	-0.620391	-1.861173	1.440057	-0.807541
Pr	0.71	-0.638521	-1.915564	1.183096	-0.502696
Pr	2	-0.638521	-1.915564	2.097062	-0.502696
Pr	7	-0.638521	-1.915564	4.063992	-0.502696
H	-0.5	-0.638521	-1.915564	1.613166	-0.502696
H	-0.2	-0.638521	-1.915564	1.509399	-0.502696

Baseline: $M = 1$, $\phi = 45^\circ$, $\beta = 0.5$, $Pm = 0.5$, $Pr = 1$, $H = 0$ and $Gw = 1$. All quantities are dimensionless except ϕ , which is in degrees. Negative Cf^* denotes resisting wall stress.

3.1 Magnetic strength and normal-component inclination

Increasing M from 0.5 to 3 increases $|Cf^*|$ from 1.825687 to 2.234109 and decreases Nu^* from 1.441909 to 1.407584 (Table 4). Figure 1 shows lower near-wall velocity and higher temperature as convection changes. The wall derivative $g'(0)$ becomes more negative, from -0.497420 to -0.521740 . Since $Bx = -Bnxag'$, the derivative and the dimensional tangential field have opposite signs for $Bn > 0$ and $x > 0$. No Joule-heating term is included.

At fixed $M = 1$, increasing ϕ from 30° to 90° raises $|Cf^*|$ from 1.825687 to 2.083230 and lowers Nu^* from 1.441909 to 1.420947 (Figure 2). Inclination enters only through $N = M \sin 2\phi$. Consequently, $M = 0.5$ with $\phi = 45^\circ$ and $M = 1$ with $\phi = 30^\circ$ produce identical reduced profiles and wall quantities.

3.2 Casson parameter and magnetic Prandtl number

Increasing β from 0.3 to 2 lowers $K\beta$ from 4.333333 to 1.5. Although $|f''(0)|$ increases from 0.523118 to 0.929321, $|Cf^*|$ decreases from 2.266846 to 1.393982 because $Cf^* = K\beta f''(0)$. Nu^* decreases from 1.464745 to 1.357088 (Figure 3 and Table 4). The velocity gradient and reduced stress therefore require separate interpretation; increasing β does not increase the effective viscosity.

Increasing Pm from 0.35 to 1 changes $g'(0)$ from -0.390857 to -0.807541 , indicating greater magnetic-field distortion. Through the coupled momentum equation, $|Cf^*|$ decreases from 1.944503 to 1.861173 and Nu^* increases from 1.431861 to 1.440057 (Figure 4). The temperature response is indirect, through the velocity field.

3.3 Thermal parameters

Increasing Pr from 0.71 to 7 thins the thermal layer and increases Nu^* from 1.183096 to 4.063992 (Figure 5). The velocity and magnetic profiles coincide because Pr enters only the energy equation. Pr is a diffusivity ratio; its variation alone does not establish a change in specific heat. Heat absorption at $H = -0.5$ lowers temperature and raises Nu^* to 1.613166, compared with 1.434848 at $H = 0$ (Figure 6). H also has no momentum or magnetic feedback.

In each magnetic panel, g' is negative at the wall and becomes positive farther away as g returns to its exterior value. It is the dimensionless magnetic-flux derivative, not the dimensional tangential field itself. The magnetic panels extend to $\eta = 35$ to show recovery, while velocity and temperature panels extend to $\eta = 12$. The computational boundary is $\eta_{\max} = 160$.

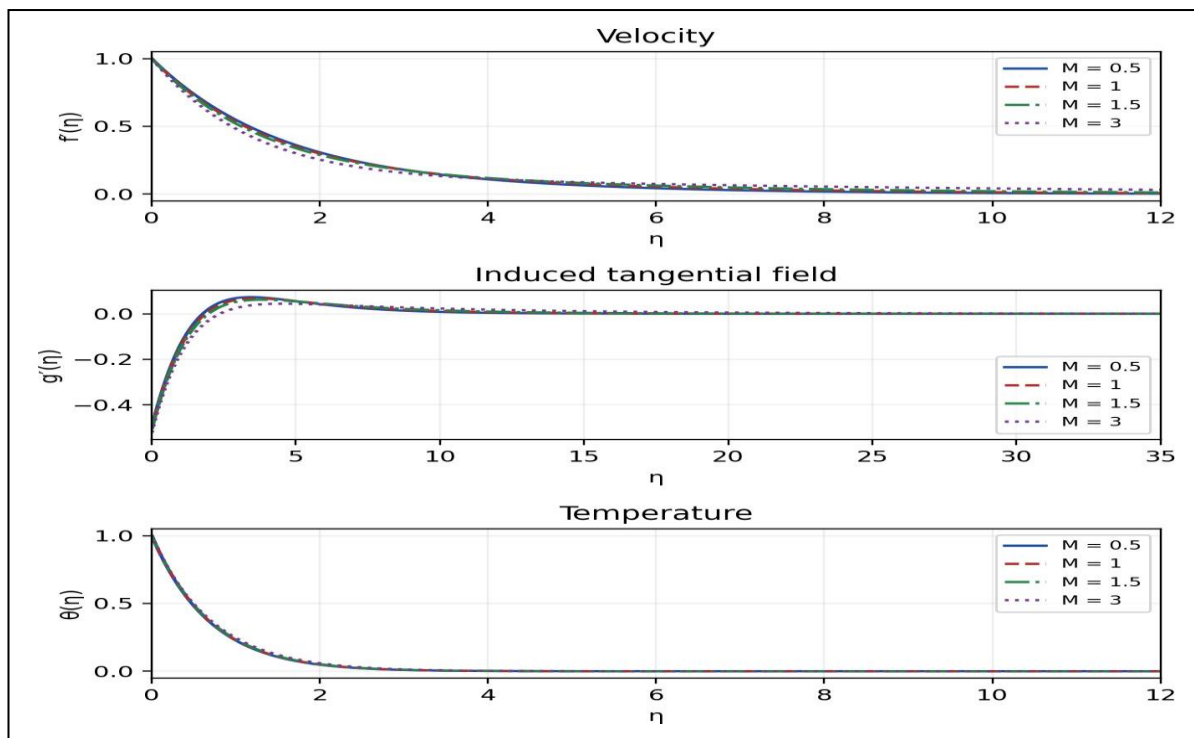


Figure 1: Magnetic-strength sweep: $M = 0.5, 1, 1.5$ and 3 . Baseline: $M = 1, \phi = 45^\circ, \beta = 0.5, Pm = 0.5, Pr = 1, H = 0$ and $Gw = 1$; only the indicated parameter varies. The top, middle and bottom panels show $f'(\eta), g'(\eta)$ and $\theta(\eta)$, respectively. The magnetic derivative satisfies $Bx/(Bnxa) = -g'(\eta)$.

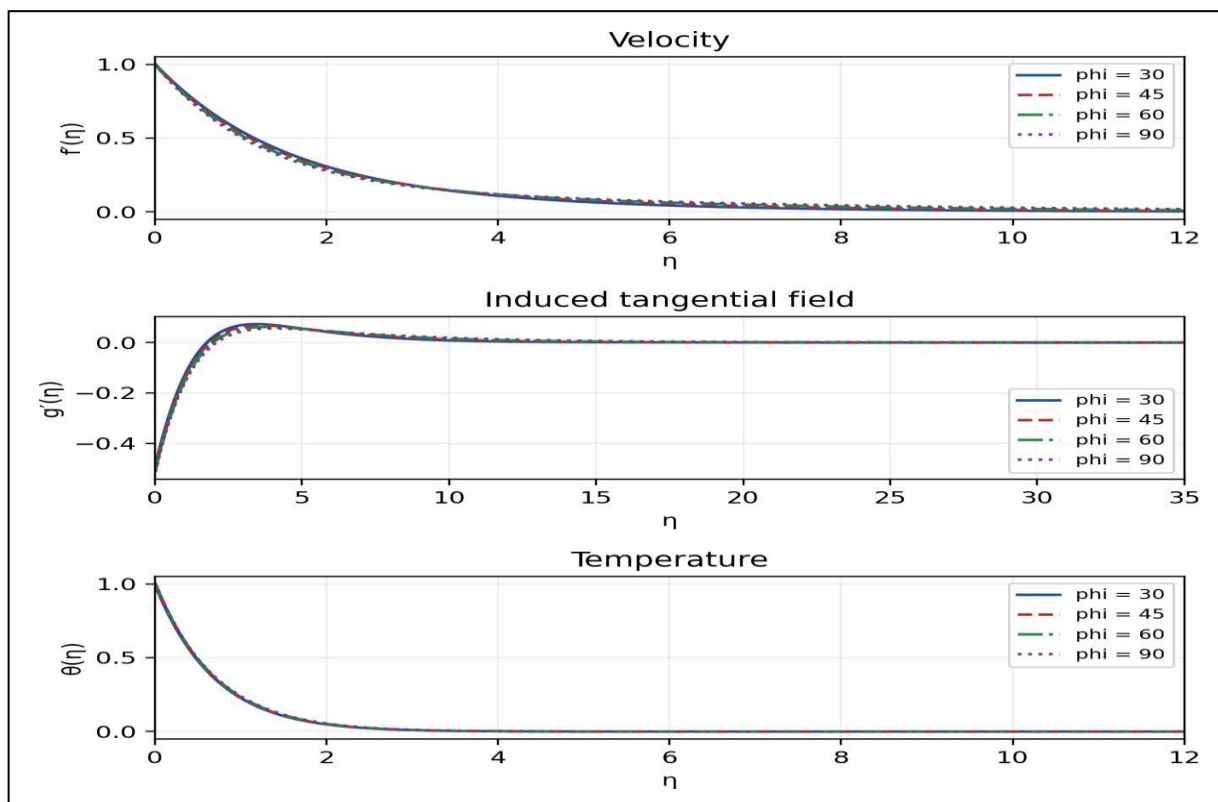


Figure 2: Inclination sweep at $M = 1: \phi = 30^\circ, 45^\circ, 60^\circ$ and 90° . Panels show $f'(\eta), g'(\eta)$ and $\theta(\eta)$. Inclination acts through $N = M \sin 2\phi$. Baseline: $M = 1, \phi = 45^\circ, \beta = 0.5, Pm = 0.5, Pr = 1, H = 0$ and $Gw = 1$; only the indicated parameter varies. The top, middle and bottom panels show $f'(\eta), g'(\eta)$ and $\theta(\eta)$, respectively. The magnetic derivative satisfies $Bx/(Bnxa) = -g'(\eta)$.

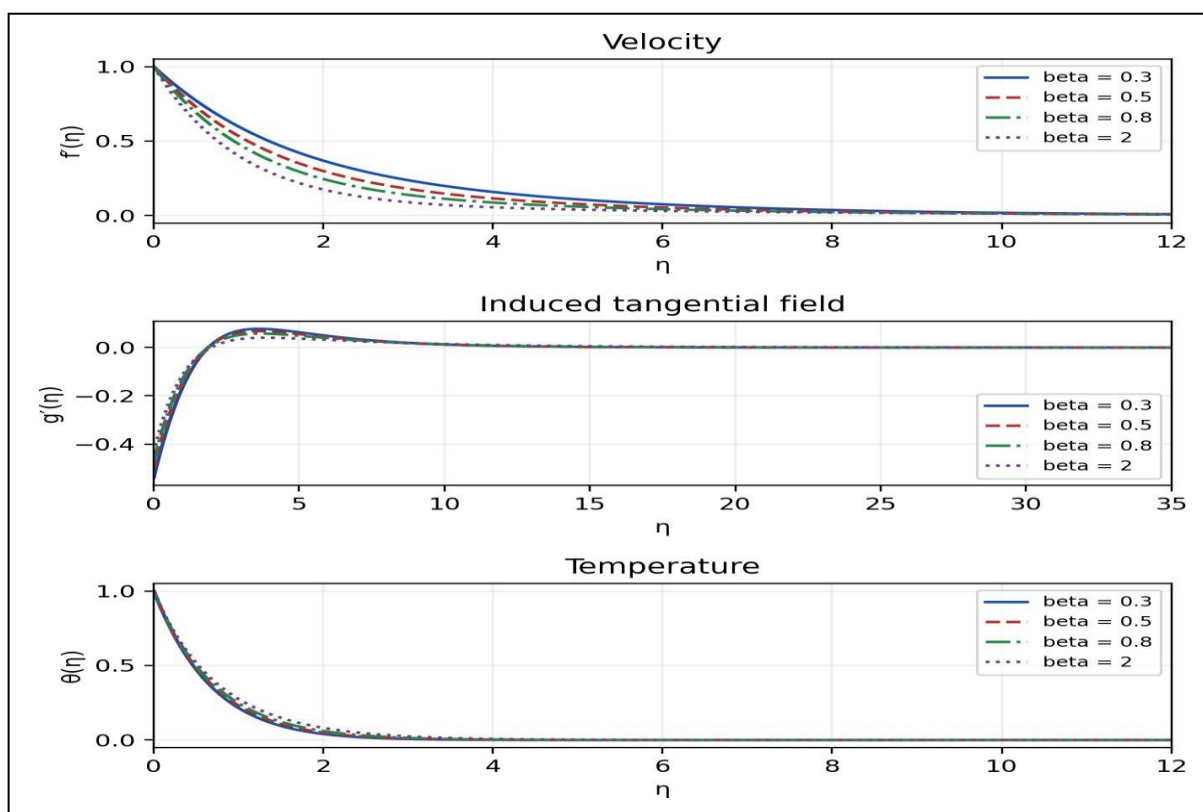


Figure 3: Casson-parameter sweep: $\beta = 0.3, 0.5, 0.8$ and 2 . Baseline: $M = 1$, $\phi = 45^\circ$, $\beta = 0.5$, $Pm = 0.5$, $Pr = 1$, $H = 0$ and $Gw = 1$; only the indicated parameter varies. The top, middle and bottom panels show $f'(\eta)$, $g'(\eta)$ and $\theta(\eta)$, respectively. The magnetic derivative satisfies $Bx/(Bnxa) = -g'(\eta)$.

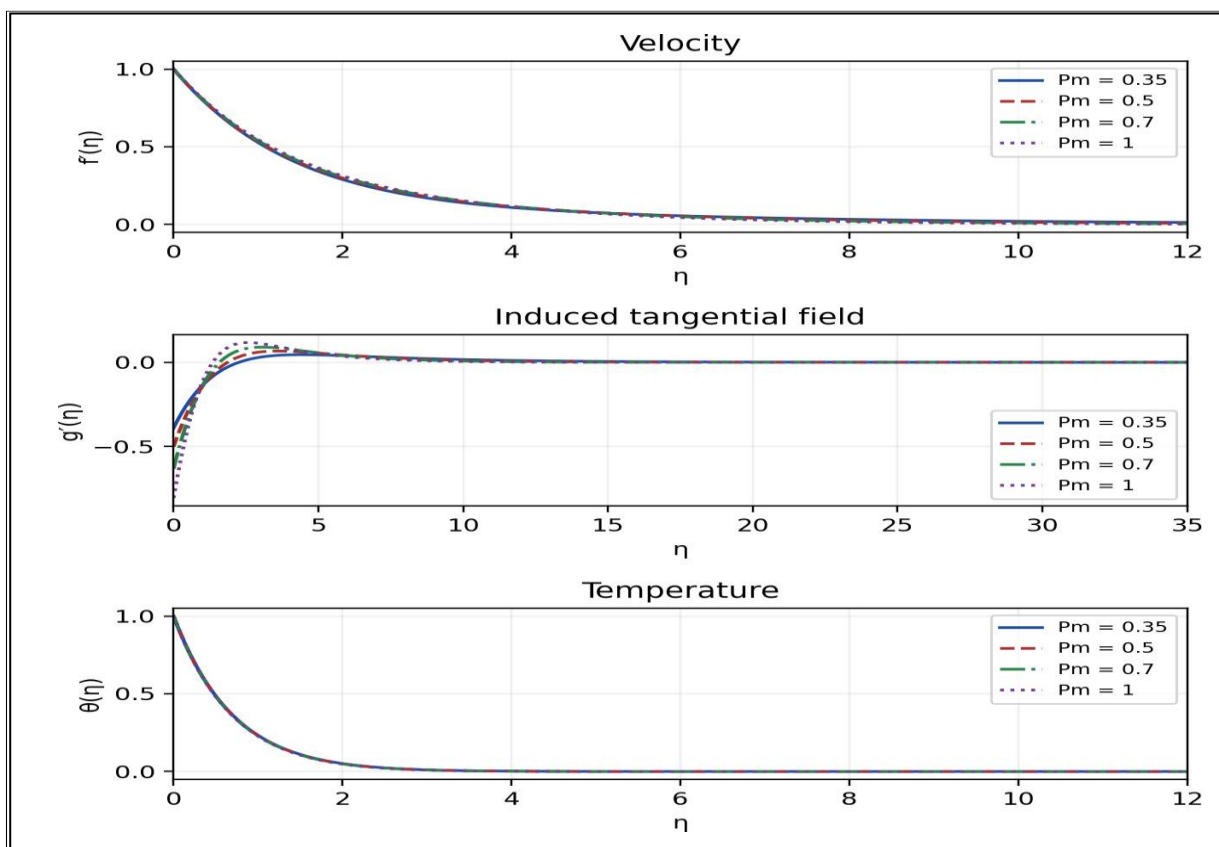


Figure 4: Magnetic Prandtl sweep: $Pm = 0.35, 0.5, 0.7$ and 1 . Baseline: $M = 1$, $\phi = 45^\circ$, $\beta = 0.5$, $Pm = 0.5$, $Pr = 1$, $H = 0$ and $Gw = 1$; only the indicated parameter varies. The top, middle and bottom panels show $f'(\eta)$, $g'(\eta)$ and $\theta(\eta)$, respectively. The magnetic derivative satisfies $Bx/(Bnxa) = -g'(\eta)$.

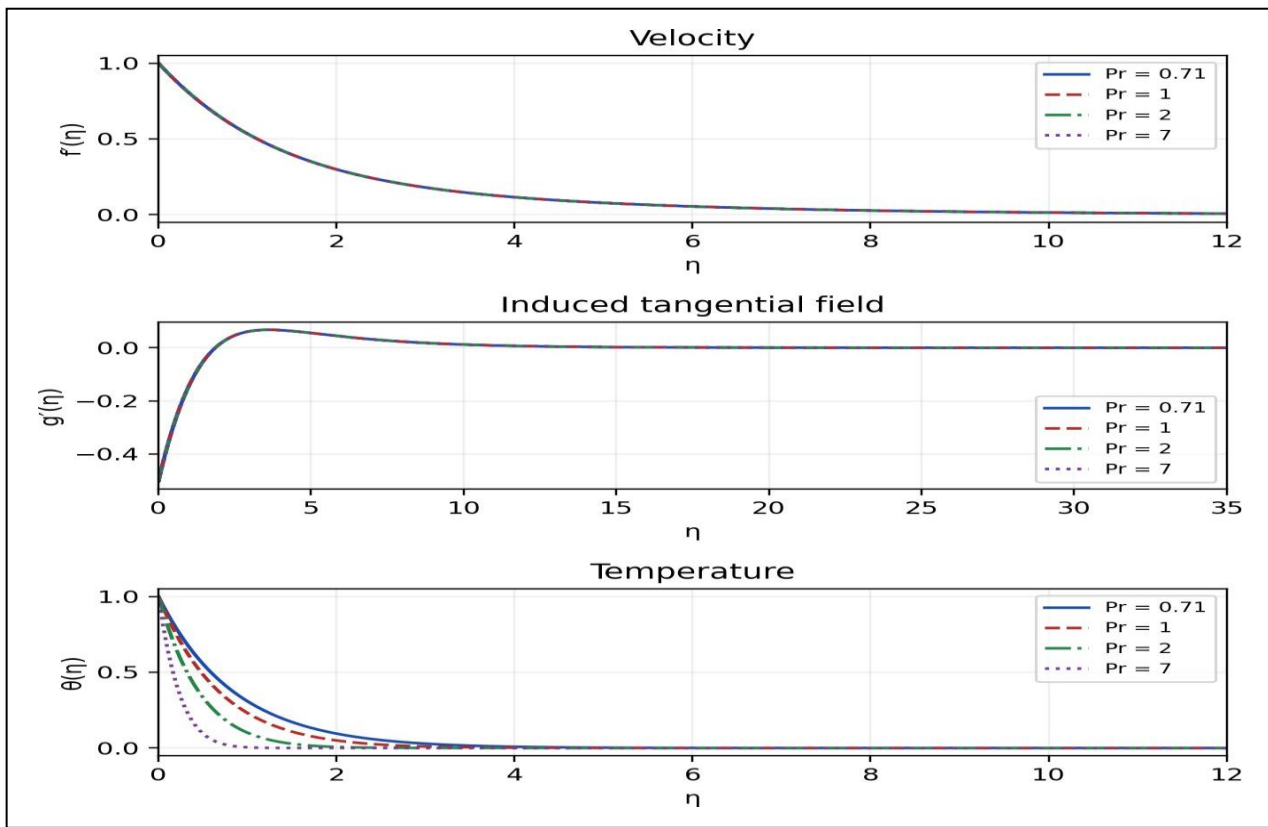


Figure 5: Prandtl sweep: $Pr = 0.71, 1, 2$ and 7 . Momentum and magnetic profiles coincide. Baseline: $M = 1$, $\phi = 45^\circ$, $\beta = 0.5$, $Pm = 0.5$, $Pr = 1$, $H = 0$ and $Gw = 1$; only the indicated parameter varies. The top, middle and bottom panels show $f'(\eta)$, $g'(\eta)$ and $\theta(\eta)$, respectively. The magnetic derivative satisfies $Bx/(Bnxa) = -g'(\eta)$.

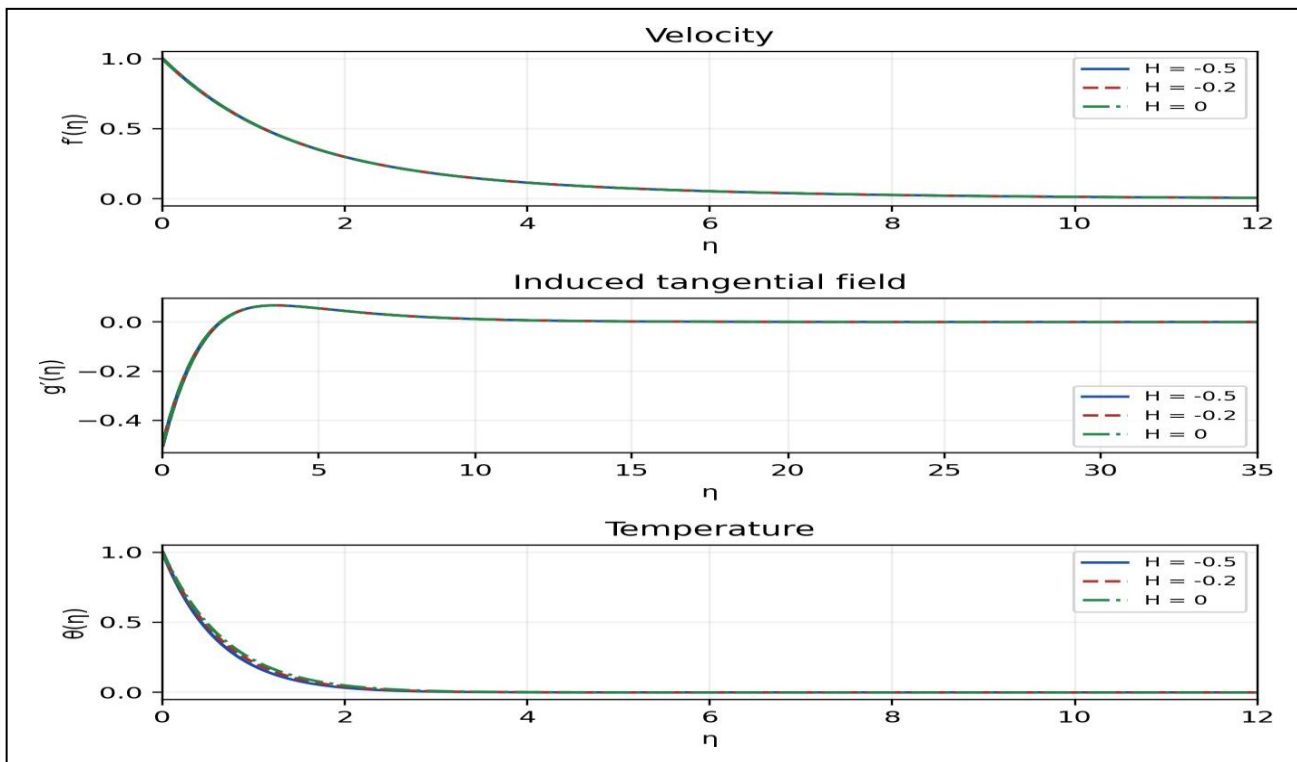


Figure 6: Heat-absorption sweep: $H = -0.5, -0.2$ and 0 . Momentum and magnetic profiles coincide. Baseline: $M = 1$, $\phi = 45^\circ$, $\beta = 0.5$, $Pm = 0.5$, $Pr = 1$, $H = 0$ and $Gw = 1$; only the indicated parameter varies. The top, middle and bottom panels show $f'(\eta)$, $g'(\eta)$ and $\theta(\eta)$, respectively. The magnetic derivative satisfies $Bx/(Bnxa) = -g'(\eta)$.



4. DISCUSSION

The calculations show how magnetic-induction feedback modifies momentum and heat transfer. Changing P_m changes both wall shear and convective heat transfer. Magnetic-pressure and magnetic-tension contributions must be interpreted together through the coupled equations. The benchmark comparisons and domain-extension checks assess numerical accuracy for the reported branch within the stated model.

Three limits constrain interpretation. First, the study is steady; no unsteadiness parameter or time-dependent performance is evaluated. Second, inclination is represented by its normal component, excluding the uniform tangential imposed field and any additional coupling associated with a complete oblique-field geometry. Third, prescribed normal flux allows a nonzero tangential field at the sheet. Replacing that condition by an insulating or conducting-wall electromagnetic condition would require a separate derivation and computation.

The Casson representation assumes a yielded fluid throughout the modelled layer and does not locate a yield surface. Because shear decays at large η , application to a material with a measured finite yield stress requires additional rheological justification. The retained magnetic Prandtl range is $0.35 \leq P_m \leq 1$; lower values were excluded because preliminary checks did not establish comparable domain-truncation accuracy. Positive heat generation lies outside the retained study and requires separate analysis of far-field admissibility. Numerical branch uniqueness and stability have not been proved.

Material-processing implications are qualitative. No experiment, dimensional material property set or device-scale optimisation is supplied. The model can compare parameter effects under its assumptions, but cannot establish engineering performance or heating caused by electrical resistance. Future work should formulate unsteady solenoidal induction, physically specified electromagnetic wall conditions, and heat dissipation before extending those claims.

5. CONCLUSION

The steady similarity formulation combines a divergence-free magnetic field with electromagnetic momentum feedback. Prescribed normal-flux boundary conditions close the seven-order system, while explicit wall definitions distinguish reduced skin friction and heat transfer from raw derivatives. The reported analytical benchmarks, thermal shooting comparison and domain-extension checks assess the computed branch. Within the sampled ranges, increasing magnetic strength or its retained normal component increases resisting reduced shear and decreases reduced heat transfer. Increasing β reduces the stress magnitude despite increasing the velocity-gradient magnitude. Magnetic Prandtl number affects velocity and heat transfer through induction feedback; Prandtl number and heat absorption affect only the thermal field. These results remain conditional on the steady, fully yielded, low-dissipation model and its prescribed normal-flux boundary.

DECLARATIONS

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Data and code availability: The supplementary reproducibility archive contains the solver source, numerical verification output, profile data and wall-quantity data used in this article.



REFERENCES

1. Abd El-Aziz, M., & Afify, A. A. (2019). MHD Casson fluid flow over a stretching sheet with entropy generation analysis and Hall influence. *Entropy*, 21(6), 592. <https://doi.org/10.3390/e21060592>
2. Crane, L. J. (1970). Flow past a stretching plate. *Zeitschrift für angewandte Mathematik und Physik*, 21, 645–647. <https://doi.org/10.1007/BF01587695>
3. Devi, R., Poply, V., & Manimala. (2020). Impact of aligned MHD flow with inclined outer velocity for a Casson nanofluid over a stretching sheet. *Heat Transfer*, 49(6), 3702–3725. <https://doi.org/10.1002/htj.21796>
4. El-Mistikawy, T. M. A. (2018). Heat transfer in MHD flow due to a linearly stretching sheet with induced magnetic field. *Advances in Mathematical Physics*, 2018, Article 5686089. <https://doi.org/10.1155/2018/5686089>
5. Kalaivanan, R., Renuka, P., Vishnu Ganesh, N., Abdul Hakeem, A. K., Ganga, B., & Saranya, S. (2015).
6. Effects of aligned magnetic field on slip flow of Casson fluid over a stretching sheet. *Procedia Engineering*, 127, 531–538. <https://doi.org/10.1016/j.proeng.2015.11.341>
7. Mahanta, G., & Shaw, S. (2015). 3D Casson fluid flow past a porous linearly stretching sheet with convective boundary condition. *Alexandria Engineering Journal*, 54(3), 653–659. <https://doi.org/10.1016/j.aej.2015.04.014>
8. Mukhopadhyay, S., De, P. R., Bhattacharyya, K., & Layek, G. C. (2013). Casson fluid flow over an unsteady stretching surface. *Ain Shams Engineering Journal*, 4(4), 933–938. <https://doi.org/10.1016/j.asej.2013.04.004>
9. SciPy community. (n.d.). `scipy.integrate.solve_bvp`. SciPy v1.17.0 reference guide. Retrieved October 11, 2026, from https://docs.scipy.org/doc/scipy-1.17.0/reference/generated/scipy.integrate.solve_bvp.html
10. Verma, V. K., & Mondal, S. (2021). A brief review of numerical methods for heat and mass transfer of Casson fluids. *Partial Differential Equations in Applied Mathematics*, 3, Article 100034. <https://doi.org/10.1016/j.padiff.2021.100034>