Efficacy of Algebraic Method in the Optimization of Linear Programming Problems: Maximization and Minimization Perspectives

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ABSTRACT

In this paper,a n algebraic method of solving optimization problems of the square matrix rectangular type is presented in place of the traditional use of the Simplex method .This algebraic method works by first converting the constraint s inequality equations into the equality type and the reducing the augmented matrix to the echelon form using row-reduction form. In this fashion the inverse of the matrix part can be obtained if the analysist want to make use of it.This method saves a lot of resources in terms of time,efforts and materials in addition to eliminating the use of fractional vales .The method is simple and straight forward and adopts the non-negativity of the variables used in the constrained inequalitity relations.The use of slack variables is omitted ,and any row with a negative value shows that the variable representing that row should not be included in the set up.The method is verified by comparing the results obtained with the ones from the simplex method from some widely used text books.In addition less computations are involved in this algebraic method where much computations take place. Problems solved here demonstrated both feasible solutions and non-feasible solutions or basic and non-basic solutions to show that the method is very effective and accurate n nature.Besides, the paper also proves that a simultaneous equation of the squared nature can be solved by converting the matrix part into an echelon form,and that one of the difficulties encountered by most stdents in the simplex method is in finding the inverse of the matrix along side with variables in the problem.The new method does not require any slack or artificial variables and does not involve taken ratios.This method also makes provisoin for the use of fewer tabular format.

Keywords: Algebraic,augmented matrix,constraints,inverse matrix, Identity matrix,optimization, slack variables.

1.0 INTRODUCTION

Operations Research (OR) is the scientific approach to decision making that seeks the optimum utilization of limited resources. In a world characterized by scarcity — limited time, limited capital, limited materials — man is constantly faced with the problem of making the best choice among alternatives (Hillier & Lieberman, 2021; Taha, 2017).

At the heart of Operations Research lies Linear Programming (LP). Linear Programming is concerned with the optimization — maximization or minimization — of a linear objective function subject to a set of linear equality and inequality constraints. Since its development by George B. Dantzig in 1947, Linear Programming has become one of the most widely applied optimization techniques in industry, agriculture, transportation, and even personal time management (Dantzig, 1963; Bertsimas & Tsitsiklis, 1997).

Two fundamental forms of LP problems dominate the literature:

1. Maximization Problems: where the objective is to obtain the highest possible value — such as maximum profit, maximum yield, or maximum spiritual growth. The standard form is typically characterized by constraints of the type :

$Ax \leq b.$ (1)

Where the terms,

A,X,and b have been defined in various literature.But in real life it means a limit where one(a buyer) is not ready to spend more!

2. Minimization Problems:* where the objective is to attain the lowest possible value — such as minimum cost, minimum time wastage, or minimum resource consumption. This is typically characterized by constraints of the type :

$Ax \geq b$ (2)

The conditions that hold in (a),also hold in (b).

In real life this is a condition where the seller of a commodity is not ready to any less than that !

While both forms seek optimality, the pedagogical challenge, especially for non-mathematicians, lies in clearly distinguishing between the desire to have _more_ and the desire to spend _less_. Yet, both pursuits are governed by the same underlying mathematical structure.

The Simplex Method, developed by Dantzig, remains the most efficient and widely taught algorithm for solving LP problems. It operates by moving from one basic feasible solution (a vertex of the convex feasible region) to an adjacent one, iteratively improving the objective function value until the optimal solution is reached (Vanderbei, 2020).

Despite the advent of interior-point methods, the Simplex Method retains its primacy due to its intuitive geometric interpretation, computational efficiency for most practical problems, and its adaptability for sensitivity analysis.The central idea behind this great feat is that given the system in an augmented matrix form,the solution to the system of equations can be optined by finding the inverse of the given matrice.Most students find this method of getting the inverse from the system to be very difficult and time coonsumming.

The purpose of this article is to (i) present a clear, non-mathematical explanation of maximization and minimization LP problems, (ii) formulate their standard mathematical models using X and Y decision variables, and (iii) demonstrate the application of the Simplex Method in obtaining optimal solutions.This method unlike the Simplex method which obtains the solution by converting the given matrix to its inverse rather converting the given matrix int an identity matrix. The paper aims to bridge the gap between theory and everyday understanding, making Operations Research accessible to students, researchers, and practitioners alike.

2. STATEMENT OF MOTIVATION / RESEARCH GAP

The motivation for this research stems from a pedagogical concern observed over years of teaching and application. While the Simplex Method remains a robust algorithm for solving Linear Programming Problems (Hillier & Lieberman, 2021), its procedural complexity — involving conversion to standard form, introduction of slack/surplus and artificial variables, construction of simplex tableau, iterative pivot operations and optimality testing — renders it tedious, time-consuming and intimidating for learners who are not mathematically gifted.

This complexity often creates a barrier to the widespread understanding and application of Operations Research principles among students of management, social sciences, and other non-mathematical disciplines. There is therefore a need for a simplified, intuitive alternative approach that preserves optimality but reduces computational burden and mathematical sophistication. This need motivated the development of the new method presented in this paper.

Contribution to Knowledge

The new method presented in this study is designed to put smiles on the faces of the Operations Research community by offering a simplistic, faster, and less mathematically demanding alternative to the conventional Simplex Method.

Unlike the Simplex Method which requires iterative tableau manipulations, the modified method:

(i) Reduces computational steps,
(ii) Eliminates the need for artificial variables in most cases,
(iii) Can be easily understood and applied by non-mathematicians,
(iv) One of the draw backs of the algebraic method is that unlike the simplex method it does not provide the inverse to the given matrix of the system of the constraints variables. Like in the simplex method the decomposition method is use is made of in solving problems with over determined/ underdetermined variables

3. METHODOLOGY

Given an objective function ,z= $x_1 + x_2 \quad x_3 + \ldots + x_n +$

Subject the constraints;

$a_{11}\ x_1 + a_{12}\ x_2 \quad a_{13}\ x_3 + \ldots + a_{1n}\ x_n \leq b_1$

$a_{21}\ x_1 + a_{22}\ x_2 \quad a_{23}\ x_2 + \ldots + a_{2n} \leq b_2$

$a_{31}\ x_1 + a_{32}\ x_2 \quad a_{33}\ x_3 + \cdots + a_{3n}\ x_n \leq b_3$

In the context of the simplex ,we are are reduce system of inequalities to the equality type:

$x_{11} \quad 0 \quad \ldots 0 = q$

$0 \quad x_{22} \ \ldots \ 0 = q_2$

……

.0 $\quad 0 \ldots\ldots x_{nn} = q_n$

4. THEOREM 1

A Linear Programming Problem problem will have a full feasible solutions if the the determinant of right hand side of the constraints exists,other wise the LPP will have an unbounded solution. the converted system

Example1.Max Z =3X +5Y

Subject

6x +5y =60

5x + 6y = 61

Solution.

We want to solve the problem by reducing the matrix part to its identity form using the tableau format;

Table 1.

Objective Z= 3X +5Y			
	x	y	
	6	5	60
	5	6	61

Table 2

Lines (l_i ,I =1,2	x	y	b
	6	5	60
($6l_2$ -5$l_1$0/11	0	1	6

If we carryout th operation, (l_1 -5l_2)/6 we get Table 3

Table 3

1	0	5
0	1	6

X =5,y =6

Hence Z = 3X+ 5Y,gives;

Z = 45

Example 2.Maxi Z = 7X + 6Y +5Z

Subject to the constraints;

X ++ 2Y + Z ≤ 8

4X - 2Y + 6z ≤ 9

3X + 4Y + 4Z ≤ 8

Solution.

Converting the system to a system of linear equations we obtain that;

X + 2Y +Z = 8

4X -2Y +6 Z = 9

3X + 4Y - Z =8

Solution.

We put the problem in the usual tabulaeu format;

Table 1

Line(LI),I=1,2,3.	x	y	z	b
L1	1	2	1	8
L2	4	-2	6-1	9
L3	3	4		8

Making only the leading diagonal element 1 on the first row to remain in column we get Table 2

Table 2

l	x	y	z	b
L1	1	2	1	8
L2-l1	0	-1	2	1
L3 -l1	0	2	-2	0

To have on 2 as the only element remainingne 2, in column 2 line 2,we carry out the operations shown in Table 3 below;

Table 3

l	x	y	z	b
L1+2l2	1	0	5	10
L2	0	-1	2	1
L3 +l2	0	0	3	2

To have 3 as the only element in column 3 of row 3,we perfom the operation as indicated in Table 4.

l	x	y	z	b
3L1-5l3	1	0	0	1
L2	0	1	0	2
2L3 -3l2	0	0	1	4

Therefore,x =1,y =2,z =4.

Since, Z = 7X + 6Y +5Z

Z = 7 +10 +20

Z = 37.

We now consider a case where we have an over determined system, more equations than the variables .

Duff p582

Find the maximum of ;

$z = 5x_1 + 4x_2 + 6x_3$

subject to :

$4x_1 + x_2 + x_3 \leq 19$

$3x_1 + 4x_2 + 6x_3 \leq 30$

$2x_1 + 4x_2 + x_3 \leq 25$

$x_1 + x_2 + 2x_3 \leq 15$

$x_1 , x_2 , x_3 \geq 0$

We set up a tabular form as shown in Table 1

Table 1

X_1	X_2	X_3	b
4	1	1	19
3	4	6	30
2	4	1	25
1	1	2	15

Here,we make use of the decomposition method by considering three submatrices,M_{123}

M_{134},and M_{234}

M_{123} Determinant =

$4x_1 + x_2 + x_3 \leq 19$

$3x_1 + 4x_2 + 6x_3 \leq 30$

$2x_1 + 4x_2 + x_3 \leq 25$

Line	x	y	z	b
L1	4	1	1	19
L2	3	4	6	30
L3	2	4	1	25

M134

$4x_1 + x_2 + x_3 \leq 19$

$2x_1 + 4x_2 + x_3 \leq 25$

$x_1 + x_2 + 2x_3 \leq 15$

M234

$3x_1 + 4x_2 + 6x_3 \leq 30$

$2x_1 + 4x_2 + x_3 \leq 25$

$x_1 + x_2 + 2x_3 \leq 15$

This is a four by three matrix,therefore we decompose it into three sub matrices

M123,M134,M234

Table 7

P = 54

Example 4 (Redundancy) Bazaraa p159

Minimize $Z = -x_1 + x_2 + x_3 = 6$

$-x_1 + x_2 + 2x_3 = 4$

$2x_2 + 3x_3 = 10$

$-x_3$ =2

$x_1, x_2 \geq 0$

Solution.0

This an over determined equation of three variables and four equations.

Tabl1

x_1	x_2	x_3	b
1	1	1	6
-1	1	2	4
0	2	3	10
0	0	-1	2

M1

Decompostion of the linear systems of equations ,we get the systems shown below;

1	1	1	6
-1	1	2	4
0	2	3	10

Determinant of M1 ≙=0

Considering x_1 , x_2

Determinat of M2 =0

Out of all the submatrices the ony that does not have a determinant of zero is the two by two matrix shown below:

2	3	10
0	-1	2

Reducing to echelon form,we get;

1	0	2
0	1	2

x_1 = 0,x_2 = 2 , x_3 =-2 but x_3 -2 negates the condition of non-negativity $x_i \geq 0$.

The ony meaniful solution is,

Z = $2x_2$ =4.

Therefore all other equations are redundant except the second equation.

Solving the systems of linear equations we have x =1,y=2,z=4.

Reducing the system to echelon form ;

X + 2Y +Z = 8

4X -2Y +6 Z = 9

3X + 4Y - Z =8

X y z

1 0 0 =

0 1 0= 2

0 0 1 = 4

Z = 7X + 6Y +5Z

Z = =39

Consider this example from H.K DASS (Advanced Engineering Mathematicspage 1277

2x +3y ≤ 12

2x + y ≤8

Solving the resulting simultaneous equations ,we find that with ,x= 3,y =2

Z = 6Y +5Z

z = 22

The same result using the laborious Simplex method!

Max z = 8x + 6y

Subject to;

8x +4y ≤ 18

X ≤ 2

Y ≤ 1.25

X > 0 *or equal to* 0

y > 0

X y

8 4 = 18

1 0 =2

0 y =1.25

Reducing the above to echelon form,,and discarding the second row for violating the non negativity condition the new table becomes

8 4 = 18

0 1 =1.25

Finally

1 0 = 1.375

He

0 1 = 1.25

Hence Z =8X + 6Y/gives

Z = 26/3

Example 4. Consider the problem from Advanced Engineering Mathematics D G Duff,p875

Max Z = 5X +9Y

Subject to the constraints:

3x + 4y ≤ 2400

x + 2y $\leq$ 900

2x + 3y $\leq$1600

2x ≤ 1200

X,y greater than or equal to zero

Setting up the system in the usual systems of linear equations:

3x + 4y = 2400

X 2y = 900

2x + 3y = 1600

2x 0 = 1 200