On Heterogeneous Vehicle Routing Problems (HVRP) with Priorities and Forced Split Delivery

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ABSTRACT

As businesses expand with numerous customers entering the supply chain daily, different cases and issues ranging from different customers with different service conditions, different vehicles with variant conditions, and the problems facing the routes are not left out. Unlike the usual splitting that occur when a vehicle can't deliver the entire quantity a customer reqiures, the main focus of this paper is to formulate a model for the Heterogeneous Vehicle Routing Problem (HVRP) that is constrained by customers' time windows, quantities, and forced split delivery. The forced split delivery herein put into consideration the splitting that is occasioned by the customers' vehicular preference, road time restriction, vehicle weight restriction, and vehicle height restriction and splitting resulting from a vehicle not being able to deliver thw entire reqest of a customer thereby necesitate .the use of two or more vehicle to service the customer. It takes into account the service choices and recent research updates on fleet sizes, mixed vehicle types, and differences in customer demand with respect to the quantity the vehicle can carry. The formulation of the HVRP multi-objective function emphasizes robustness and demonstrates a simplified architecture that intertwines minimization of the total travel cost as it relates to the travel schedules, the travel cost, or the interspace, minimization of the running cost as it relates to the fixed cost as well as the variable cost of several channels used, maximization of the clienteles' priorities, and lastly, carters for the marginal difference in vehicle capacity. The paper stresses the formulation of the multi-objective with a view to picture real-life business scenerios.

Keywords: Vehicle Routing Problem (VRP), Forced Split Delivery, Road Restrictions, Heterogeneous Vehicle Routing Problems (HVRP), Customers' Priorities.

INTRODUCTION

The traditional Vehicle Routing Problem (VRP) is targeted at exploring a set of outlined paths that begin at the depot and terminate the route at the depot to determine the minimum cost (determining the closest path, reducing the number of vehicles, etc.) such that the known request of all the customers is met. Every customer is traditionally allowed to be visited once by just one vehicle, and each vehicle's carrying capability is defined. However, economic globalization has led to a swiftly growing movement of commodities and services all around the world. The meager commodities coupled with the transportation resources, the complications faced in planning, and the increasing cost strain resulting from keen rivalry among the logistics service providers make use of computer-aided systems crucial for the robust, efficient, and adequate planning of transportation systems. A salient sub-task in this frame of reference is the operational arrangement and design of vehicles or other specialized transportation means called VRP.

Thousands of papers with enormous variants of VRP written over the years indicate the pragmatic and theoretical relevance of this NP-hard optimization problem to solve vast problems in this area. Therefore, many unique solvers for distinct VRPs can be instituted in the literature. The hitch is that most of the methods used are highly inflexible and specific, so it needs a lot of tools to adapt to modified problems.

The astounding advancements in information technology warrant companies to concentrate on the cost-effectiveness, proficiency, and timeliness of the supply chain. With this, the VRP has become a priceless tool in fashioning different designs and operations in the supply chain. Essential VRP applications include routing delivery by school buses, industrial refuse collection, bank vans, urban newspaper distribution, security patrol services, supermarkets, and postal/mail deliveries. The development of broadened and in-depth research directed at solving this problem is owing to its increased practical visibility and applicability.

Moreover, most real-life problems are generally much more entangled than the idealized problems in research, and they change over time with different customers, vehicles, and routes. To circumvent these issues, we give a fused modeling and optimization framework for solving multiplex and practical relevant VRP that considers intermittencies in VRP, the inflow of priorities, and restrictions on the road to be traversed. Some formulations in the literature have presented constraints on the maximum traveling time. The VRP with Split Delivery (VRPSD) is a variant of the traditional VRP where customers can be visited by one or more vehicles. Consequently, aside from the number to be delivered by each vehicle and the delivery routes to each customer for the VRPSD, we hope to stress Forced Split Delivery caused by road restrictions. However, the requisite objectives of VRP are to determine the minimum amount of vehicles, the minimum travel time, and the minimum total costs over the traveled routes.

Usually, splitting is done when the quantity that a customer requires is more than the quantity the vehicle can carry. This paper will open a vista of knowledge on different categories of splitting and a keen search for other reasons why there could be forced splitting. The concept of forced split delivery is such that results from splitting are not only as a result of the quantity that the customer requires exceeding the quantity the vehicle can carry but also consequential on the customers' vehicular Preference, Road Time restriction, Vehicle Weight Restriction, or Vehicle Height Restriction.

Heterogeneous Vehicle Routing Problems (HVRP)

Heterogeneous vehicles involve vehicles with varying capacities, fuel costs, and fixed costs. This category of routing problems, which are frequent in logistic operations, takes into consideration the situations in our real-life settings where it is practically not possible to claim to have only one type of vehicle for distribution, knowing fully well that there are several reasons why different types of vehicles are needed.

There are many grounds on which distribution managers will want to use a fleet of vehicles of varying capacities, sizes, fuel costs, and fixed costs. Such rationale for HVRP includes:

1. traffic flow regulated for specific times, weight restriction, vehicle height control, etc. The peculiarity of road conditions made it necessary for companies to use non-identical types of vehicles to service customers' requests in good time;
2. While large vehicles are routinely cost-effective on long distances, they are sometimes disallowed to ply some routes in metropolises owing to environmental degradation, road maintenance, and traffic congestion, or they may not be used to visit some particular customers owing to certain restrictions on the road infrastructures.

The idea of HVRP was first borne in [1]. This generalization is salient in pragmatic terms since most customers' demands might not have been met by one single type of vehicle; hence, the customers are served by several vehicles as in [2] and [3]. A typical HVRP incorporates the time dependency of traveling at a specific time of the day. The authors in [4] opined that the target of HVRP is to minimize the total costs of traveling and reduce the elapsed time over the entire planning time frame.

The goal of the HVRP is to determine the most appropriate composition of the fleet of vehicles and fashion out a routing plan corresponding to minimize the total cost of transportation. Owing to different assumptions, there are several versions of HVRP in the literature. These assumptions are mainly associated with and answer the following questions:

- is there a fixed cost for each vehicle type;
- each type of vehicle has a limited number;
- the cost of routing for different types of vehicles is not the same;
- each customer has a type of vehicle so preferred.

Herein, the authors take into consideration all the aforementioned assumptions.

Fundamental Requirements of HVRP

To enable one spells out the fundamental requirements for HVRP, obviously, there is a necessity to examine VRP over a given frame of time, T. Let $C = \{c_i \mid i = 0, 1, 2, \ \dots\ , \ N\}$ represent N set of customers such that c_0 is the depot. Let $V = \{V_k \mid k = 1, 2, \ \dots, M\}$ denote M sets of a heterogeneous fleet of vehicles positioned at the depot, where $V_1 = \{v_1^a \mid a = 1, 2, 3, \dots, A\}$, $V_2 = \{v_2^b \mid b = 1, 2, 3, \dots, B, \ \dots, V_M = \{v_M^z \mid z = 1, 2, 3, \dots, Z\}$ be the subsets of vehicles in each heterogeneous type of vehicle in set V, where $A, B, \dots, Z$ denote the number of vehicles in each heterogenic set. By [5], every deuce of locations, (i, j), of two successive nodes affirmed as customers, with $i, j \leq N$. Associated with $i \neq j$, is the travel time, t_{ij}, from one customer c_i to the next customer,c_{i+1}, and the distance traveled by the vehicle, $d(i, j) = d_{ij}$, is symmetrical, i.e., $d_{ij} = d_{ji}$.

The following fundamental requirements are binding on every customer, c_i:

- there should be a pre-determined quantity, $q(c_i)$, of the commodities or services that the customer requires to be delivered. The dispatch manager then decides which of the heterogeneous vehicles, M, will be most appropriate for the delivery. Note, not in all cases can a vehicle do the entire delivery. However, where one vehicle cannot do the entire supply required by a customer, then the supply has to be split. The splitting will be done either among different types of vehicles, $V_1, V_2, V_3 \ \dots \ V_M$, or among the same types of vehicles, $v_1^1, v\,_1^2, v_1^3, \ \dots\ , \ v_1^A$.
- there should be a set-out time, t_{ij}, required by the vehicle, V, to traverse from either the depot, c_0, to the customer, c_i, or from one customer, c_i, to the next customer, c_{i+1}, to unload the quantity, $q(c_i)$. It ultimately leaves the customer, c_{i+1}, for either the next customer, c_{i+2}, or returns to the base station if every customer along the path that the vehicle is assigned has been attended to or, summarily, the vehicle carriage quantity, $Q(V_k)$, has been exhausted.
  Another way around, there could be situations where more than one vehicle of the same type, v_1^1, $v_1^2, v_1^3 \ \dots \ v_1^A$, or more than one vehicle of different types, $V_1, V_2, V_3 \ \dots \ V_M$, have to serve a customer. In such cases the vehicles move directly to the customer right from the depot. The time, t_{ij}, required by the vehicles either moving from the base station, c_0, or from a particular customer, c_i, to the next customer, c_{i+1}, to discharge the quantity, $q(c_j)$, and ultimately leave for another customer, or invariably return to the base station must be specified.
  The priority, δ, of the customer, $\delta(c_i)$, is satisfied by the vehicle, V, and must be stated clearly. Every customer is serviced from only one depot with a heterogeneous and finite fleet of vehicles. The set of vehicles, V, with quantities each different vehicle can carry is represented by $Q(V_k)$.

  The following characteristics are peculiar to the vehicle:

- there is a fixed working period for the vehicle, $T(V_k)$, with the earliest or starting time, $T^s(V_k)$, and the finishing time, $T^f(V_k)$ ie $T^f(V_k) - T^s(V_k) = T(V_k)$

- there is a fixed cost, $FC(V_k)$, for each vehicle, which is the wages of the driver and the loaders attached to each vehicle.
- the quantity the vehicle can carry, $Q(V_k)$, must be known at the depot.

Leaning on the layout requirements for the customers as well as the fundamental characteristics of the vehicles, the following assumptions are derived thus:

- the variable cost, VC_{ij}, is the servicing cost from one customer c_i to the next customer c_{i+1};
- the time, t_{ij}, is the traveling time from one customer c_i to the next customer c_{i+1}. If all the road/traffic restrictions remain the same, the traveling time is assumed to be symmetrical, $t_{ij} = t_{ji}$,;
- $R_i = \{r_i(1), \dots, r_i(S)\}$ stands for the set of routes for the vehicles, V, the number of customers serviced along a route is represented by S, and $r_i(S)$ represents the ith customer visited. It is also assumed that every route terminates at the depot hence, $r_i(S+1) = 0$;
- the distance from where the vehicle parks to discharge to the warehouse/store of each customer is assumed equal hence, unloading time per unit is kept constant.

The next section shall discuss the priorities as a ground to usher in the HVRP objective function formulation.

Customers' Priorities

Recently, with the growing population, variation in customers' requests, and the various priorities which customers place on their demand, have led to a corresponding increase in the number of vehicles e.g. time of delivery, quantity to be supplied, dynamical situation to be considered, e. t. c. are few of such priorities.

A customer can desire that more than one of the priorities have to be met. However, this work will accommodate only two priorities: priority based on time and the priority based on quantity.

Priority Based on Time

Time priority is the commonest priority that customers place on vehicle routing. Priority based on time is correlated with the customers in a time sphere (see [6]), in which each customer has to be visited. In VRPTP, every customer sets out a time-space in which the servicing must begin and, in some cases, when it must end. The VRPTP can be used to structure different practical applications. The authors in [5] itemized some such real-life situations.

The authors in [7] presented an amplified time priority that takes into consideration multiple periods. It ascertains that every clientele is called on as a result of a given frequency and given attainable combinations of visiting periods. Current practical use includes scheduling and routing elevator preventive maintenance service teams allocated to various customers' locations as discussed in [8] and the Austrian Red Cross periodic blood products delivery to hospitals as opined by [9]. For coherence, the time windows have been subdivided into four. Each customer, $c_i \in N$, is assumed to have a time window that falls within an interval $(t_n^e,\ t_n^l)$ i.e., $t_n^e < t_{ij} < t_n^l$, (see [10] and [11], which coincide appropriately with the earliest time t_n^e, and latest time t_n^l, that the vehicle will visit the customer, c_i.

According to [5], [12], [13], and [10], let $PT_a(c_i)$ denote the time priority type a where $1 \le a \le 4$ for customer c_i; hence the following scenarios arise:

$PT_1(c_i) = t_1^e < t_{ij} < t_1^l$: The period space depicts that the vehicle can come in any time after the earliest time, t_1^e, and must depart the customer's place before the latest time. This time window, $(t_1^e,\ t_1^l)$ allows the vehicle to visit the customer at a convenient time within the working duration, T_k.

$PT_2(c_i) = t_2^e < t_{ij} \leq t_2^l$: In this time-space, the vehicle can reach the customer's spot after the earliest time but must surely depart before or at the nick of the latest departure time.

$PT_3(c_i) = t_3^e \leq t_{ij} < t_3^l$: This time frame allows the vehicle to come in on or after the earliest time and depart any time before the latest departure time. The earliest time is said to be closed, while the latest time is not. In this case, if the vehicle arrives ahead of the arrival time, it cannot discharge; hence, it has to wait till the earliest arrival time.

$PT_4(c_i) = t_4^e \leq t_{ij} \leq t_4^l$: This is a case of boundedness at the arrival and departure times. The vehicle may arrive at a time on or after the earliest time but must depart on or before the latest departure time.

It must be noted that every customer can make a choice of any of the four priorities, but only one of the time priorities is allowed i.e.,

$PT_a(c_i) = PT_1(c_i), PT_2(c_i), PT_3(c_i)$, or $PT_4(c_i)$. (1)

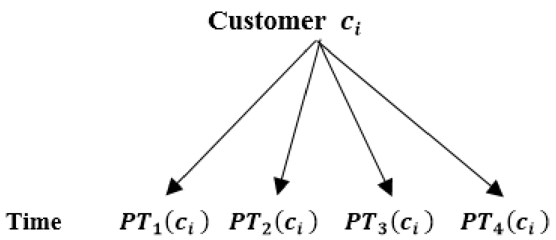


Figure 1: Time Priority Tree.

From Figure 1, a customer, c_i, can only be allowed to be linked to a time priority space.

Priority Based on Quantity

Two different cases of priority based on quantity will be enumerated here as observed by [5], [14], and [15] thus:

(i) where the quantity a customer requests falls within the quantity the vehicle can carry. Visiting such clientele along the path leads to specifying the quantities without any win-win situation. Every customer, c_i, has an amount demanded that falls within the interval $[q^{min}, q^{max}]$, corresponding respectively to the customer's minimum quantity and maximum quantity demanded (see [16]).

(ii) where the quantity a customer requests may only be supplied by more than one vehicle. This happens when the quantity a customer requires is more than the carrying quantity of the vehicle as given in (6) and (8).

Here, one considers priorities based on the quantity to be supplied as stipulated by customers. Since the quantity required is a range, the priorities is expressed as intervals with (PQ_i^{min}, PQ_i^{max}), $1 \leq i \leq 4$. Hence

$PQ_1 = q_1^{min} < q(c_i) < q_1^{max}$: This indicates that the quantity that the customer is requesting is greater than q_1^{min} but less than q_1^{max}.

$PQ_2 = q_2^{min} < q(c_i) \leq q_2^{max}$: The quantity required by the customer is greater than q_2^{min} but less or equal to q_2^{max}.

$PQ_3 = q_3^{min} \leq q(c_i) < q_3^{max}$: Here, the amount a customer will be supplied with can be q_3^{min} or more than q_3^{min} but must be less than q_3^{max}.

$PQ_4 = q_4^{min} \leq q(c_i) \leq q_4^{max}$: Here, the quantity the customer requires this happens all the time $[q_4^{min}, q_4^{max}]$. It is strictly bounded, with a possibility of q_4^{min} equal to q_4^{max}. But practically, there are situations when a customer might need more commodities due to an urgent need. Such cases do not allow the customer to vary its demand even when it could be at its disadvantage to both the customer and the supplier.

It must be emphasized that, usually in practical situations when the interval is closed at both ends, it is not always flexible enough and not an ideal priority. This is because it does not allow for business expansion. Also, only one of the priority placed by the customer is allowed as illustrated in the tree diagram.

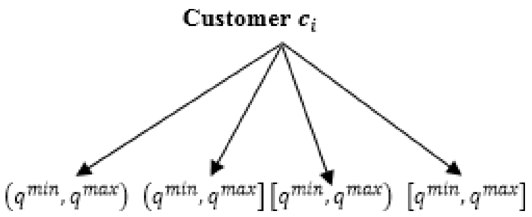


Figure 2: Quantity Priority Tree

Figure 2 shows that a customer, c_i, can only be linked to one quantity priority.

$PQ_b(c_i) = PQ_1(c_i), PQ_2(c_i), PQ_3(c_i)$, or $PQ_4(c_i)$ (2)

where $PQ_b(c_i)$ stands for the customer's priority. It must be put on record that each customer is tied to having priorities based on time and priority based on quantity. Whenever the quantity a customer requires is above the quantity that the vehicle can carry, then such is faces with forced split deliveries; otherwise, split deliveries are not always needed.

Split Delivery

In practical situations, irrespective of whether the vehicles in use are homogeneous or heterogeneous, split deliveries usually happen when the request of a customer cannot be satisfied by one vehicle. Findings have shown that there are cases whereby the demand of customers outweighs the carrying capacity of vehicle allocation. Since, by VRP design and formulation, a vehicle is not permitted to service a customer twice on the same day, split delivery becomes necessary.

The reasons for split deliveries could be that:

(a) the quantity requested by a customer, $q(c_i)$, exceeds the quantity the vehicle can carry $Q(v_k^a)$. Given by the relation:

$$q[c_i(v_k^a)] > Q(v_k^a) \qquad (3)$$

This would lead to the delivery being done by more vehicles of the same type (homogeneous):

$SD_1(c_i) = Q(v_k^a) + Q(v_{k+1}^a) - q[c_i(v_k^a)] \geq q(c_i)$ (4)

On the other hand, the delivery could be done by different vehicles (heterogeneous):

$SD_2(c_i) = Q(v_k^a) + Q(v_k^{a+1}) - q[c_i(v_k^a)] \geq q(c_i)$ (5)

where $Q(v_{k+1}^a)$ and $Q(v_k^{a+1})$ are the carrying quantities of vehicles v_{k+1}^a and v_k^{a+1} respectively, $q[c_i(v_k^a)]$ is the amount supplied or to be supplied to customer c_i, $SD_1(c_i)$ and $SD_2(c_i)$ are split deliveries while

$[Q(v_{k+1}^a)$ or $Q(v_k^{a+1})]$ appropriately, is the splitting quantity to be delivered by the same or different type of vehicle respectively to the same customer c_i.

(b) If the vehicle, v_k^a, has serviced some customers say, $c_1, c_2, \ldots, c_{N-n}$, along the route, $r_i\,(S_i\,)$, with the quantities, $q(c_1), q(c_2), \ldots, q(c_{N-n})$, where $n < N$ denotes the number of customers that have been visited on that path $r_i\,(S_i\,)$. The quantities delivered by the vehicle v_k^a to the customers $c_1, c_2, \ldots, c_{N-n}$, are given by:

$$q[c_1(v_k^a)] + q[c_2(v_k^a)] + \cdots + q[c_{N-n}(v_k^a)]$$

$$= \textstyle\sum_{i=1}^{N-n} q[c_i(v_k^a)]. \qquad (6)$$

Then, the quantities to be conveyed by the vehicle v_k^a to the next customer c_{N-n+1}, is given by: $q[c_{N-n+1}(v_k^a)]$ and the total amount to be delivered by vehicle v_k^a is given by:

$$\textstyle\sum_{i=1}^{N-n} q[c_i(v_k^a)] + q[c_{N-n+1}(v_k^a)]$$

$$= \textstyle\sum_{i=1}^{N-n+1} q[c_i(v_k^a)]\,. \qquad (7)$$

With (7), if the leftover quantity $q[c_{N-n+1}(v_k^a)]$, which would have been delivered to the next customer, c_{N-n+1}, along the same route, is not sufficient i.e.,

$$\textstyle\sum_{i=1}^{N-n+1} q[c_i(v_k^a)] > Q(v_k^a) \qquad (8)$$

then, it occasions a split delivery to be undertaken by another vehicle of the same type, v_{k+1}^a, as:

$$\{\textstyle\sum_{i=1}^{N-n} q[c_i(v_k^a)]\} + q[c_{N-n+1}(v_{k+1}^a)] \qquad (9)$$

or another vehicle of a different type, v_k^{a+1} as:

$$\{\textstyle\sum_{i=1}^{N-n} q[c_i(v_k^a)]\} + q[c_{N-n+1}\,(v_k^{a+1})]. \qquad (10)$$

From (9) and (10), the split quantity, $q[c_{N-n+1}\,(v_{k+1}^a)]$ or $q[c_{N-n+1}\,(v_k^{a+1})]$ required by c_{N-n+1} is the remaining quantity to have been delivered by v_k^a, which is:

$$Q(v_k^a) - \sum_{i=1}^{N-n} q[c_i\,(v_k^a)] + q[c_{N-n+1}\,] =$$

$$q[c_{N-n+1}\,(v_{k+1}^a)] \text{ or } q[c_{N-n+1}\,(v_k^{a+1})] \qquad (11)$$

Since the quantity that will be delivered by $Q(v_{k+1}^a)$ or $Q(v_k^{a+1})$ cannot be determined a priori then:

$$\boldsymbol{SD_3(c_i)}$$

$$= \boldsymbol{Q(v_k^a) + Q(v_{k+1}^a) + \cdots + Q(v_M^a) - \textstyle\sum_{i=1}^{N-n} q[c_i\,(v_k^a)] = \sum_{k=1}^{M} Q[(v_k^a)] - \sum_{i=1}^{N-n} q[c_i\,(v_k^a)] \geq q[c_{N-n+1}\,]}$$
(12)

Or alternatively,

$SD_4(c_i)$

$$= Q(v_k^a) + Q(v_k^{a+1}) + \cdots + Q(v_k^A) - \sum_{i=1}^{N-n} q[c_i\,(v_k^a)]$$

$= \sum_{a=1}^{A} Q[(v_k^a)] - \sum_{i=1}^{N-n} q[c_i\,(v_k^a)] \geq q[c_{N-n+1}]$ (13)

A more complex case is when (12) and (13) cannot meet the quantity required by c_{N-n+1}, leaving us with the option of having:

$$Q(v_k^a) + Q(v_{k+1}^a) + \cdots + Q(v_M^a) + Q(v_k^{a+1}) + \cdots - \sum_{i=1}^{N-n} q[c_i\,(v_k^a)] \geq q[c_{N-n+1}]$$

$SD_5(c_i) = \{\sum_{k=1}^{M} Q[(v_k^a)]\} + Q(v_k^{a+1}) + \cdots - \sum_{i=1}^{N-n} q[c_i\,(v_k^a)] \geq q[c_{N-n+1}]$ (14)

Conversely, we have

$$Q(v_k^a) + Q(v_k^{a+1}) + \cdots + Q(v_k^A) + Q(v_{k+1}^a) + \cdots - \sum_{i=1}^{N-n} q[c_i\,(v_k^a)] \geq q[c_{N-n+1}]$$

$SD_6(c_i) = \{\sum_{a=1}^{A} Q[(v_k^a)]\} + Q(v_{k+1}^a) + \cdots - \sum_{i=1}^{N-n} q[c_i\,(v_k^a)] \geq q[c_{N-n+1}]$ (15)

It is clear from above that a customer that the split delivery condition applies to can be linked to only one of either (4), (5), (12), (13), (14), or (15).

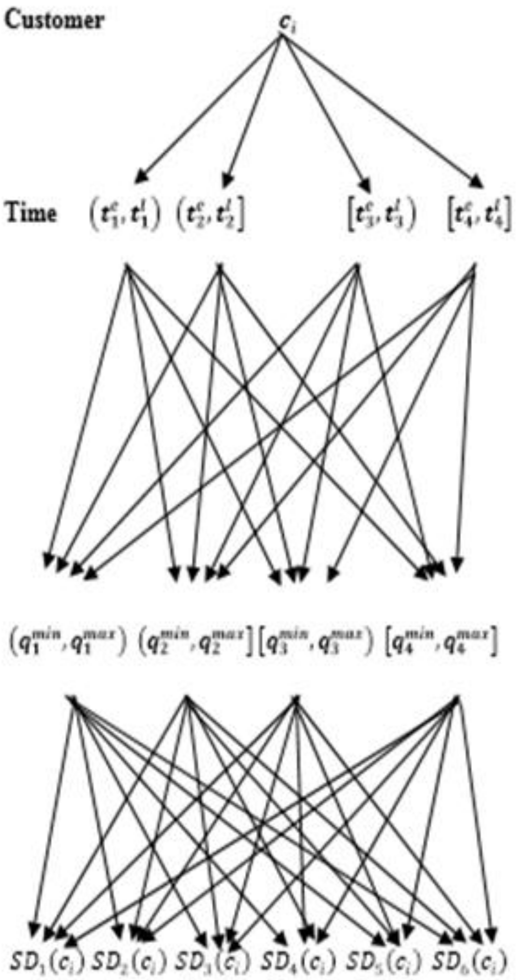


Figure 4: Priorities Interplay Tree

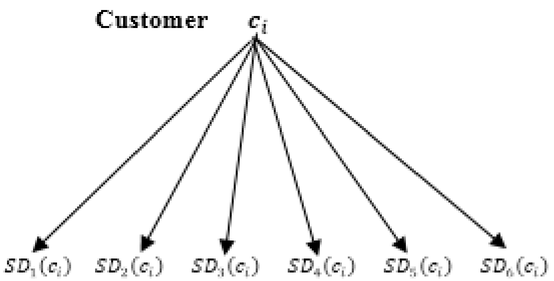


Figure 3: Split Deliveries Tree

where $SD_c(c_i)$ represents the split delivery assigned to a customer. Attention is drawn to the fact that not every customer falls into a split delivery situation. Since every customer can only fulfill one set of time and quantity priority conditions, the various priority interplays shall be enumerated via the tree diagram in Figure 4.

From the priority interplay tree shown in Figure 4, it becomes clear that, when a customer, c_i, is not visited, the Time Priority of the customer, $PT_a(c_i) = 0$, and the Quantity Priority of the customer, $PQ_b(c_i) = 0$ else, $PT_a(c_i) = 1$ and $PQ_b(c_i) = q(c_i)$. Hence, the sum of all the time and quantity priorities for all the customers $c_1, c_2, \dots, c_N$ is given by:

$$\sum_{i=1}^{N} \delta(c_i) = \sum_{i=1}^{N} \sum_{a=1}^{4} \sum_{b=1}^{4} PT_a(c_i) \times PQ_b(c_i) \quad (16)$$

In addition, if the amount demanded by the customer, $q(c_i)$ is beyond the carrying quantity of the vehicle or has failed to satisfy all the road restriction conditions (to be discussed in subsequent sections) then, $PQ_b(c_i)$

will require split delivery, $SD_c(c_i)$. When there is splitting, the sum of the time spaces and the amount required by the customers is given by:

$$\sum_{i=1}^{N} \delta(c_i) = \sum_{i=1}^{N} (\sum_{a=1}^{4} \sum_{c=1}^{6} PT_a(c_i) \times SD_c(c_i)) \quad (17)$$

Road Restriction

Transportation plays an important role in our day-to-day activities. The advancement of transport-related energy, congestion, consumption, and its adverse impact on our surroundings have drawn more global concerns. An increasing number of vehicular flows on the highway has led to traffic jams, pollution, road degradation, and lots more. Traffic planners hence tend to put restrictions on highways, ranging from one reason to the other. Therefore, continuous concerted efforts on VRP will not only be directed towards reducing the transportation cost but contribute to our environmental protection.

Some roads are more susceptible to damage than others based on poor drainages, weather conditions, and other variables. Based on the weather conditions and frost testing bans are pronounced. Road bans are effective tools for preventing road damage, reducing maintenance costs, and ensuring roadways remain safe for all motorists.

Road restrictions may be categorized in the following groups:

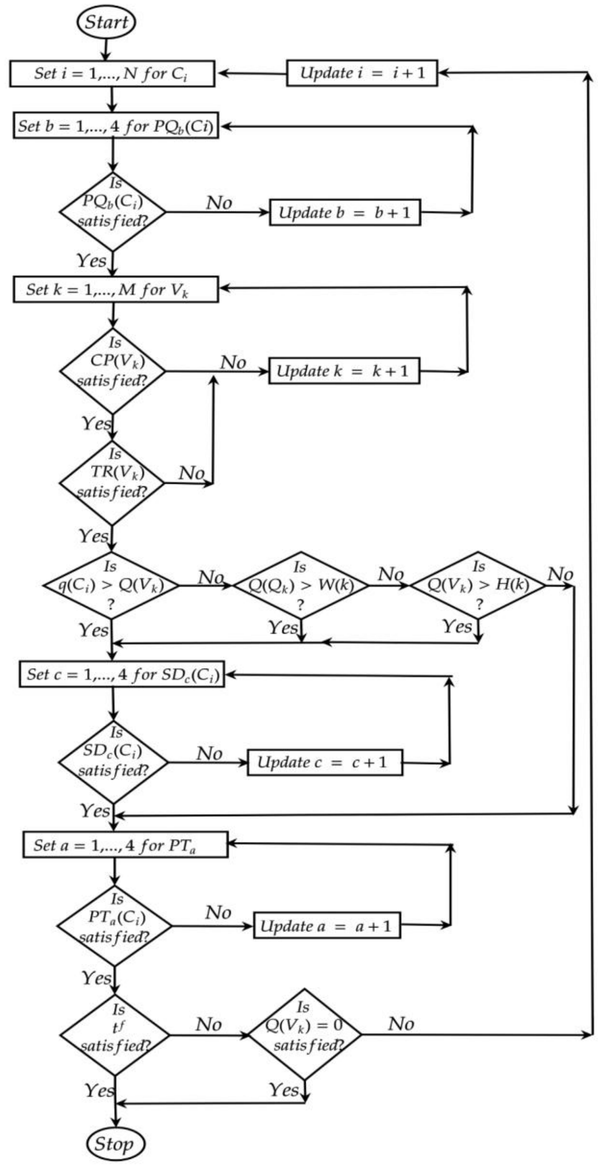
- Road Time Restrictions, $TR(V_k)$: The $TR(V_k)$ disallows the movement of vehicles at a particular time in some particular places and restricts the movements of some types of vehicles in some routes to reduce the traffic in such areas.
- Vehicular Weight Restriction, $WR(V_k)$: The $WR(V_k)$ is placed on some roads to regulate the weight of vehicles that traverse such roads purposely to keep the road infrastructure from further damage or total breakdown.
- Vehicular Height Restriction, $HR(V_k)$: Not all roads allow any height of the vehicle. Height restrictions are placed to regulate either heavy-duty vehicles or vehicles with too high consignment from plying particular roads to avoid degradation, total damage, or accidents on the road.
- Customer's Preference for a vehicle, $CP(V_k)$:

When a road ban occurs, signs indicating the allowed axle percentages are posted, and the ban is monitored and enforced to ensure compliance. Such restrictions are not for life rather, the authority in charge fixes them. Once the road repair has been done and declared structurally okay for use, the ban can be rescinded. Until recently, split delivery has been a function of a vehicle not being able to supply all the quantity a particular customer requires.

However, it has become obvious that there could be forced split delivery which results from situations where: there is the Customer's Preferred Vehicle to do the delivery; there is Road Time Restriction; Vehicle Weight Restriction limiting the carriage of the vehicle and there is Vehicle Height Restriction limiting the height of the consignment that the vehicle can carry irrespective of whether the load is light.

In a case where a vehicle fails to satisfy all four road restriction conditions, ultimately, the delivery will have to be split among vehicles. Such splitting is referred to as Forced Split Delivery.

While the next section will dwell on the introduction of the priorities into the HVRP objective function so formulated, it is expedient to look at the connectivity between the road restriction conditions and the priorities



as it dovetail to forced split delivery with the flowchart that follows.

Figure 5: Flowchart Showing the Connectivity Between Road Restrictions and Priorities.

Formulation of the HVRP Objective Function

Time Windows and Quantities priorities that came up in practical situations in VRPSD is a multi-objective function. Here, J_1 calculates the distance of the customer or the carrying cost to each of the customers, J_2 is directed at determining the vehicle's fixed cost and J_3 is aimed at solving for the priorities of the customers.

If the vehicle v_k^a has to visit customer c_{i+1} just after visiting customer c_i then, $\xi_{ijk} = 1$ otherwise, $\xi_{ijk} = 0$. The distance traveled or carrying cost is given as: $Min\, J_1 = \alpha \sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} d_{ij}(c_j)\xi_{ijk}$ (30)

and the fixed cost is given as:

$$Min\, J_2 = \beta \sum_{k=1}^{M} \sum_{i=0}^{N} \sum_{j=1}^{N} FC(v_k^a)\xi_{ijk} \qquad (31)$$

If (3) holds, $FC(v_k^a)$ in (16) becomes

$$FC(v_k^a) = FC(\sum_{k=1}^{M} Q[(v_k^a)] - \sum_{i=1}^{N-n} q[c_i\,(v_k^a)])$$

Or

$$FC(\sum_{a=1}^{A} Q[(v_k^a)] - \sum_{i=1}^{N-n} q[c_i(v_k^a)]). \qquad (32)$$

One then obtain

$$J_2 = \beta \sum_{k=1}^{M} \sum_{i=0}^{N} \sum_{j=1}^{N} FC(\sum_{k=1}^{M} Q[(v_k^a)] - \sum_{i=1}^{N-n} q[c_i\,(v_k^a)])\xi_{ijk} \qquad (33)$$

Or

$$J_2 = \beta \sum_{k=1}^{M} \sum_{i=0}^{N} \sum_{j=1}^{N} FC(\sum_{a=1}^{A} Q[(v_k^a)] - \sum_{i=1}^{N-n} q[c_i(v_k^a)])\xi_{ijk} \qquad (34)$$

Or

$$J_2 = \beta \sum_{k=1}^{M} \sum_{i=0}^{N} \sum_{j=1}^{N} FC(\{\sum_{k=1}^{M} Q[(v_k^a)]\} + Q(v_k^{a+1}) + \cdots - \sum_{i=1}^{N-n} q[c_i\,(v_k^a)])\xi_{ijk} \qquad (35)$$

Or

$$J_2 = \beta \sum_{k=1}^{M} \sum_{i=0}^{N} \sum_{j=1}^{N} FC(\{\sum_{a=1}^{A} Q[(v_k^a)]\} + Q(v_{k+1}^a) + \cdots - \sum_{i=1}^{N-n} q[c_i\,(v_k^a)])\xi_{ijk} \qquad (36)$$

$Max\, J_3 = \gamma \sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} \delta(c_i)\xi_{ijk}$ (priorities) (37)

where α, β, and γ are specified constants for weighting corresponding terms to J_1, J_2, and J_3 as opined by [1], and A stands for any of the set in $\{A,\ B, \ldots ,\ Z\}$.

On combining the three sub-objectives (30), (31), and (37) for cases where there is no split delivery, one obtains:

$$J = J_1 + J_2 + J_3 \qquad (38)$$

$$J = \alpha \sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} d_{ij}(c_i)\xi_{ijk}$$

$$+ \beta \sum_{k=1}^{M} \sum_{i=0}^{N} \sum_{j=1}^{N} FC(v_k^a)\xi_{ijk}$$

$$+ \gamma \sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} \delta(c_i)\xi_{ijk} \qquad (39)$$

However, when split delivery is involved, one combines the three sub-objectives: (30), only one of (33), (34), (35), or (36) as appropriate instead of (31), and (37) according to the peculiarity of the problem.

The following constraints apply:

$\sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} \xi_{ijk} \leq 1,\ j = 1, \ldots, N$ (40) $\sum_{i=0}^{N} \sum_{p=1}^{N} \sum_{k=1}^{M} \xi_{ipk} - \sum_{p=1}^{N} \sum_{j=2}^{N} \sum_{k=1}^{M} \xi_{pjk} = 0,$ (41)

$p = 1, \ldots, N$ and $i \neq j = 1, \ldots, N$ $\sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} q(c_i)\xi_{ijk} \leq Q(v_k^a),$ (42)

$$\sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} t_{ij}\,\xi_{ijk} \leq T^f(v_k^a) - T^s(v_k^a), \qquad (43)$$

$$\sum_{i=0}^{N} \sum_{j=1}^{N} \sum_{k=1}^{M} \xi_{ijk} \leq 1, \qquad (44)$$

$$y_i - y_j + N\sum_{i=0}^{N}\sum_{j=1}^{N}\sum_{k=1}^{M}\xi_{ijk} \leq (N-1) \qquad (45)$$

$\xi_{ijk} \in \{0,1\}\ \forall\ i,\ j$ and k (46)

The constraint (40) stresses a customer can only be serviced by a vehicle at most once a day. Constraint (41) states that a vehicle that visits a customer must leave the customer's place either for another customer or back to the depot. Constraint (42) expresses the quantity a vehicle can carry on each route.

However, where $q(c_i) > Q(v_k^a)$, a splitting will be necessary. The constraint (43) is the working duration of each vehicle on each route. Constraint (44) states that a vehicle can only be used at most once a day. Where y_i is an arbitrary constant, and by [17] the relation (45) is the sub-tour-elimination condition attached to the Travelling Salesman Problem (TSP) and VRP as opined by [18] and [19]. This is aimed at forcing each of the routes to pass through the depot. Constraint (46) is the integrality conditions.

If the focus of the performance measure of the VRP is mainly to determine the priorities then, the first and the second series in (39) will be set as zero. If the performance measure of the VRP aims to determine the priorities as well as the traveling cost alone then the second series in (39) is set as zero. But where it is aimed at calculating the priorities, traveling cost, and the fixed cost then, (39) holds.

CONCLUSION

This paper presents a solution to Supply Chain Management and Distribution problems currently ravaging businesses in the world. It is aimed at formulating an objective function that takes into consideration forced split deliveries as it concerns road time restrictions, vehicle weight restriction, and vehicle height restriction, which opens a vista of knowledge on HVRP as well as looking into the differences in customers' demand over vehicle carrying quantities leading to splitting of deliveries.

The formulation of a road restricted HVRP objective function with forced split deliveries emphasizes robustness, efficiency, and demonstrates a simplified architectural function that minimizes the total traveling cost, minimizes the operational cost, maximizes the customers' preferences and carter for the marginal difference that occurs in vehicle carrying quantity.

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